Provider-driven complementarity and firm dynamics*

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Abstract

Business dynamism has experienced a significant decline during the last decades in the U.S. This paper offers a new explanation based on the assumption of provider-driven complementarity, which makes seemingly independent products become complements when provided by a single firm. I develop a quality ladder growth model where provider-driven complementarity is crucial in determining firms' incentives to challenge incumbents in their established markets. I show that a decline in the average size of innovations induces a growth slowdown. Moreover, I find that the entry rate declines, and both concentration of sales and Research and Development expenditure increase even as the growth rate of the economy declines. This is in contrast to a standard quality ladder model without provider-driven complementarities which implies the reverse. The asymmetry generated by provider-driven complementarity between potential entrants' and incumbents' R&D incentives is key for the decline in business dynamism.

JEL Codes: E23, L22, O31.

Keywords: Growth, research and development, declining business dynamism, provider-driven complementarity, concentration, market power.

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1 Introduction

The recent debate on increasing firm concentration and profits highlights some troubling facts regarding declining business dynamism and competition in the United States. This literature documents that nowadays markets are more concentrated and less competitive than they were decades ago.[1] Adding to this literature, this paper reviews a series of trends on declining business dynamism using data from Compustat, Business Dynamics Statistics, and World Development Indicators. These trends are as follows:

1. The entry rate of new firms has declined.
2. Market concentration, measured by the share of sales accruing to the biggest firms, has increased.
3. Expenditure on R&D activities, measured both as a fraction of total cost or total sales, has increased.
4. Productivity growth has slowed down.

Taken together, while the entry rate of new firms has been declining, the so-called 'superstar' firms have become bigger and more profitable.[2] This in turn has raised concerns regarding dominant firms crowding out new entrants and reducing entrepreneurship. At the same time, economic growth has been sluggish during the last decades,[3] though R&D efforts have increased substantially.

In this paper, I propose a theoretical framework that explains increasing R&D expenditures and concentration yet decreasing entry rates and economic growth. The key novelty of the model is introducing provider-driven complementarities into an otherwise standard quality ladder model. Provider-driven complementarity makes initially independent products become complements when provided by a single firm. It boils down to the idea that during the process of product innovation – the introduction of new and improved products to the market – firms can incorporate differential characteristics to their products. These firm-specific characteristics, which can be associated with the brand, software, or product design, are such that, absent quality differences across products, consuming several goods from a single provider is preferable to purchasing each good from a different firm.

Theoretically, I build on the Akcigit and Kerr (2018) model of endogenous growth through R&D. The economy is formed by a representative household and an endogenous measure of firms. Each firm owns a product portfolio that supplies monopolistically to the market. All firms have access to the same production technology, and product quality grows on a ladder through stochastic quality arrivals arising from investment in R&D. Firms' R&D is of two types: internal (improve the quality of a product within its portfolio) and external (improve the quality of a product outside its portfolio). I model provider-driven complementarity as a demand shifter embedded in the production process,

[1] See, among others, Autor, Dorn, Katz, Patterson and Van Reenen (2020), or Akcigit and Ates (2019*a*) for the first feature, and (Duernecker, Herrendorf and Valentinyi, 2019), (De Loecker, Eeckhout and Unger, 2020), (Shambaugh, Nunn, Breitwieser and Liu, 2018) or (Feijoo Moreira, 2020) for the second.

[2] In line with Autor et al. (2020), the term 'superstar' refers to the most productive firms in an industry.

[3] See (Duernecker et al., 2019), or (Gordon, 2018) for some examples on growth slowdown.

increasing in the number of (different) products supplied by the same firm.[4] Therefore, upon entering the economy, any firm is ex-ante able to generate the same complementarity level. In other words, there exist only two sources of heterogeneity across firms that evolve endogenously as a result of innovation: the number of products in their portfolio and the quality of each product.

In a standard quality-ladder model, successful R&D improving the quality of a variety enables the innovator to price-out a lower-quality incumbent. However, when firms generate provider-driven complementarity, consumers do not necessarily switch to the state-of-the-art highest quality product. Instead, they may remain attached to the lower-quality incumbent if the provider-driven complementarity derived from this firm is sufficiently large. In equilibrium, each market is supplied by its market leader: the firm able to offer the highest quality, adjusted by provider-driven complementarity, relative to its market price. This bears an important effect on R&D decisions: when there exists provider-driven complementarity, the size of the quality improvement that an innovator requires to become a market leader depends not only on the size of its product portfolio but also on the size of the product portfolio of the incumbent. In particular, the probability of obtaining an innovation that allows replacing an incumbent – labeled a successful innovation – is a function of the product portfolio size. Specifically, firms with large portfolios are ex-ante more likely to obtain successful innovations than smaller ones. Put differently; smaller firms need to obtain larger quality innovations than bigger firms to offset the provider-driven complementarity effect of a given incumbent. Therefore, firms conduct R&D for two reasons: i) it allows increasing their market share by selling higher quality goods and/or capturing more markets, and ii) it increases the provider-driven complementarity effect as firms increase their product portfolio. As a result, firms' R&D decisions affect the industrial organization of firms and can ultimately deter firm entry, because provider-driven complementarity generates an endogenous barrier to entry in new markets. In other words, a key novelty of the provider-driven complementarity framework is that the equilibrium distribution of products across firms affects firms' R&D decisions, which in turn affect aggregate variables.

I use the theory of provider-driven complementarity to perform a quantitative exercise in which I reduce the size of the average quality jump stemming from any successful innovation. This exercise is motivated by the recent literature on ideas becoming harder to find, in the spirit of Bloom, Jones, Van Reenen and Webb (2020), and can also be thought of as innovations becoming less radical over time.[5] The reduction in the average innovation step size mechanically generates a slowdown in the economy's growth rate. Most importantly, it introduces rich dynamics in the R&D decisions of firms when they generate provider-diver complementarity. By targeting the decline in the U.S. growth rate, I show that there is less entry while incumbents become bigger and spend more resources on R&D, even as the economy's overall growth rate declines. This contrasts with the predictions of a standard quality ladder model without provider-driven complementarities, which implies the reverse.

The interaction between innovative step size and provider-driven complementarity in

[4] This is equivalent to assuming that product complementarities are inherent to consumers' preferences and independent of the production technology of firms.

[5] The observation of slowing technical change goes back to the mid-1960s and 1970s, as does the idea of 'exhaustion of inventive opportunities', see Griliches (1994) for a review.

determining which firm supplies each product in equilibrium is key to the previous results. When the average innovative step size declines, the probability of obtaining a successful innovation also does. Moreover, this has a direct implication on the rate of creative destruction. As it turns out, this rate – which is a decreasing function of a firm's number of products – declines as innovators find it more difficult to come up with successful innovations. Therefore, all else equal, small firms – and mainly potential entrants – find it more difficult to become market leaders. In particular, more quality innovations that would be successful in the absence of provider-driven complementarity do not find their way into the markets. The decline in the probability of obtaining a successful innovation can be broken down into two components. The first one is mechanical: reducing the average step size innovation makes firms less likely to obtain successful innovations. The second component is the change in the distribution of firms that affects firms' incentives to conduct R&D. As a result, firms' industrial organization matters for equilibrium outcomes if firms generate provider-driven complementarity, a novel and crucial feature of this framework.

Additionally, the reduction in the step size of innovation affects incumbents and potential entrants asymmetrically in the provider-driven complementarity framework. This results from the interaction between two forces: the *market effect* and the *quality effect*. The *market effect* captures the increase in the value of the discounted stream of profits associated with being a market leader, which enhances the incentives to conduct R&D. In equilibrium, the interest rate and the rates of creative destruction decline, and so does the effective discount rate of firms profits. The *quality effect* captures the decline in the productivity of investing in quality. When the step size of innovation declines, firms find it more difficult to come up with successful innovations and obtain smaller quality improvements if successful. Both effects decrease the incentives to conduct R&D. I show that when the step size of innovation declines, incumbents conduct less internal R&D (the *quality effect* dominates the *market effect*) and more external R&D (the *market effect* dominates the *quality effect*).

As a consequence, this leads to an overall increase in the R&D expenditure of incumbent firms. However, potential entrants – that only conduct external R&D and do not generate complementarity upon entry – conduct less R&D due to the decline in the probability of obtaining a successful innovation (the *quality effect* dominates the *market effect*). This decline drives down the entrants' innovation rate, which reduces the entry rate of new firms. The joint effect of the decline in entry and the increase in external R&D innovation rates of incumbents is a reduction in the number of active incumbents in equilibrium. Accordingly, the equilibrium firm size distribution shifts to the right as a substantial share of firms become bigger. In turn, this yields an increase in the concentration of sales.

Literature review. This paper relates to two strands of the economic literature. First and foremost, to the recent and growing literature on declining business dynamism starting with (Decker, Haltiwanger, Jarmin and Miranda, 2016). There are many contributions to this literature that offer various explanations for increased market concentration and declining business dynamism. Aghion, Bergeaud, Boppart, Klenow and Li (2019) investigate whether falling firm-level costs of spanning multiple markets due to accelerating IT advances can explain the trends observed in the data. The authors show that as the cost of spanning into multiple markets declines, the most efficient firms expand into new markets while less efficient firms find it more difficult to enter profitably, innovating less. Moreover, while a temporary surge of growth occurs in the short run, in the long run, innovation and productivity growth decline as both high and low productivity firms' incentives to conduct R&D are hampered because incumbent firms tend to be high productivity firms. Akcigit and Ates (2019*b*) analyze a series of margins that shape competition dynamics and ultimately show that a decline in the intensity of knowledge diffusion from frontier firms to laggards can explain rising market concentration and a slowdown in business dynamism. Liu, Mian and Sufi (2020) find that low interest rates can explain rising concentration and declining dynamism by encouraging investment for industry leaders relative to its followers. Cavenaile, Celik and Tian (2020) find that the increase in markups is mainly driven by a decrease in competition from small firms, but among large firms. The authors show that markups have actually contributed to productivity growth, and estimate that the increase in the cost of innovation is the fundamental reason underlying the productivity slowdown. Peters and Walsh (2019), Hopenhayn, Neira and Singhania (2018), Engbom (2020), Bornstein (2018), and Röhe and Stähler (2020) provide different explanations that relate aging and declining population growth with declining entry rates of new firms and increased concentration. Closest to this paper are De Ridder (2020) and Olmstead-Rumsey (2020). On the one hand, De Ridder (2020) shows that the trends in productivity growth, R&D expenditure, and business dynamism can be explained by an increase in firms' use of intangible inputs. On the other hand, Olmstead-Rumsey (2020) shows that average patent quality has fallen over the same period and can explain the aggregate productivity growth slowdown and market concentration increase. I contribute to this literature by offering an additional explanation based on the role of provider-driven complementarity, a simple mechanism that shifts demand as a function of firm size. Contrary to these two papers, the model I propose does not rely on ex-ante firm heterogeneity. However, the model can successfully explain many empirical findings on business dynamism – e.g., the joint observation of increasing R&D effort and decreasing economic growth – through the asymmetries that provider-driven complementarity generates between small and big firms.

Second, this paper is tightly related to the literature on (quality ladder) endogenous growth models. The main contributions to this literature are the seminal papers of Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992), and the Schumpeterian growth models developed in Klette and Kortum (2004), Lentz and Mortensen (2008), and Acemoglu and Cao (2015) where a single firm operates each product line as a monopolist. My model builds upon the recent and influential Akcigit and Kerr (2018) framework. Although these models usually feature an endogenous distribution of products across firms, this distribution is a crucial equilibrium element in my theory of provider-driven complementarity. Specifically, firms' industrial organization is a determinant not only of individual firm decisions, but also for aggregate outcomes. A key novelty of the provider-driven complementarity framework is that the probability of obtaining a successful innovation that allows entering a market, and the rate of creative destruction experienced by an incumbent, crucially depend on the number of products that each firm supplies to the market. Ultimately, this implies that the distribution of products across firms is not a residual equilibrium outcome like in most growth models, but a key element that affects firms' optimal R&D decisions. Although this substantially complicates the model's solution, the modeling choice of provider-driven complementarity remains tractable and allows obtaining analytical solutions.

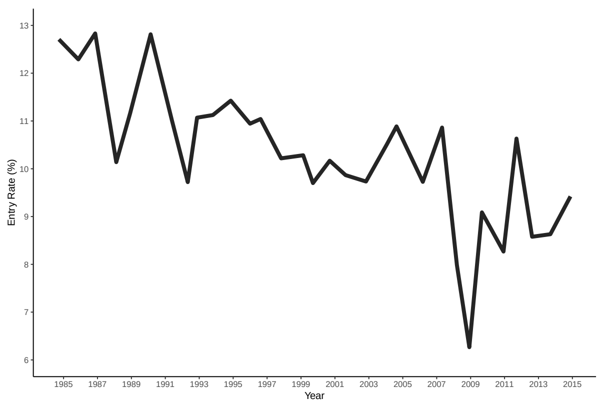
Layout. The rest of this paper is organized as follows. In Section 2, I briefly review some recent key empirical findings on firm dynamics in the U.S. In Section 3, I develop a novel theory of provider-driven complementarity. In Section 4, I carry out my main quantitative experiment and discuss the relevance of the asymmetries generated by provider-driven complementarity for the results. I additionally provide a sensitivity analysis of the results. In Section 5, I conclude.

2 Business dynamism, concentration, R&D expenditure, and provider-driven complementarity

In this section, I briefly discuss the empirical trends that motivate this paper. The analysis is based on several facts that have already been documented in the literature. The empirical trends I show use data from Compustat, Business Dynamics Statistics, and World Development Indicators.[6] I also provide suggestive evidence of the relevance of provider-driven complementarity in shaping consumer demand.

Fact # 1: Declining entry rate. I start by reviewing a well-known and widely documented fact about declining business dynamism in the United States. Figure 1 shows the firm entry rate using data from the BDS. Since 1985, the firm entry has contracted in around 1/3, declining by 5 p.p. As Decker et al. (2016) and Akcigit and Ates (2019*a*) show, the same trend holds for establishments. This decline in the entry rate is also related to the long-run decline in the startup rate Karahan, Pugsley and Şahin (2019).

Figure 1: Firm entry rate

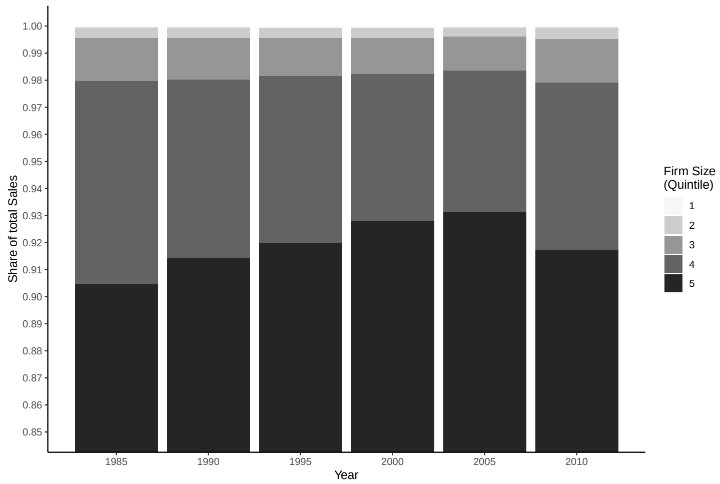


Source: Author's calculations using Business Dynamics Statistics data.

[6]Specific details about the data can be found in Appendix A.1.1.

Fact # 2: Increased firm concentration. An additional well documented fact is the contemporaneous increase in the concentration of businesses (see, among others, (Akcigit and Ates, 2019*b*), (De Loecker et al., 2020), (Grullon, Larkin and Michaely, 2018), (Van Reenen, 2018), (Aghion et al., 2019), or (Helpman and Niswonger, 2020)). Figure 2 shows the five-year average share of sales of each quintile of firms in Compustat. From 1985 to 2010, the share of sales held by the 20% biggest firms in the U.S. economy increased from roughly 90% of the total sales between 1985 and 1990 to 93% between 2005 and 2019. From 2010 to 2015, this share has declined slightly in the aftermath of the Great Depression.

Figure 2: Cumulative share of sales by firm quintile



Source: Author's calculations using Compustat data.

Fact # 3: Increased R&D expenditure. The endogenous growth literature stresses innovation – typically the result of successful R&D activities – as a key driver of aggregate growth. Figure 3 depicts the total expenditure in R&D as a share of total cost[7] or total sales for firms in Compustat. During the last decades, R&D expenditure has roughly doubled relative to firms' total cost and total sales.[8]

Fact # 4: Growth slowdown. Through the lens of the standard growth literature, the apparent increase in R&D effort would lead to stronger aggregate growth. However, the last decades have been characterized by stagnant or even declining growth rates.[9] Figure 4 shows the slowdown of GDPpc growth. While in the mid-1980s, GDPpc growth averaged 2.5%, this rate has declined to 1.5% in the mid-2010s.

[7]In the data, the total cost of firms is Operating Expenditure (OPEX), the sum of Cost of Goods Sold (COGS), and Selling, General, and Administrative Expenses (SGA).

[8]The increase in R&D can also be observed by using aggregate data from the OECD. For example, the share of Gross domestic expenditure on R&D has also increased during the last decades. The interested reader can find additional details in Appendix A.1.2.

[9]See, for example, Duernecker et al. (2019), Gordon (2018) or Gordon and Sayed (2018).

Figure 3: R&D expenditure as a fraction of Total Sales and Total Cost

Source: Author's calculations using Compustat data

Figure 4: U.S. Real GDPpc Growth

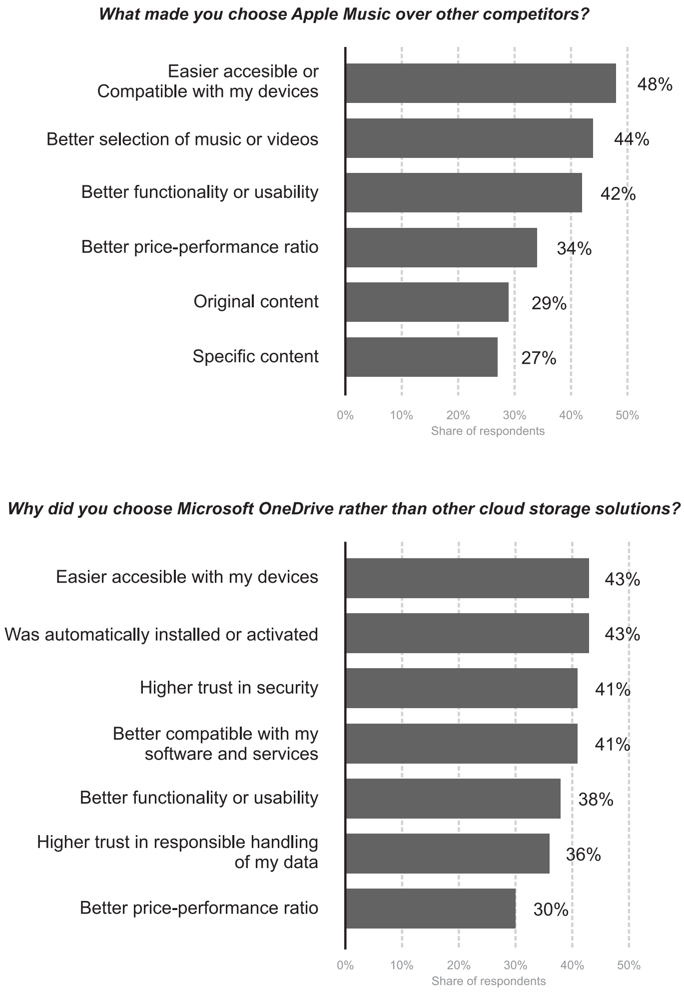
Source: World Development Indicators.

Provider-driven complementarity. Figure 5, shows the responses to two questions regarding the consumption decisions of two well-known internet-based services: Apple Music and Microsoft OneDrive.[10] Both questions try to address what is the most relevant aspect shaping consumers' consumption decisions. As it turns out, both services are designed in a way that are easier to access/are better compatible (read-as more complementary) with other goods or services provided by the same firm. More strikingly, this is a more important factor in shaping demand than the price-performance (read-as price-to-quality) ratio.

Based on this suggestive evidence, in this paper, I model the fact that consumers value more products when the same firm offers them. This novel mechanism, labeled provider-driven complementarity, is defined as firms' ability to transform seemingly independent products into complements when provided by a single firm. Ultimately, provider-driven complementarity is a firm-specific characteristic that is incorporated into its products during the process of product creation and innovation. The next section develops a theoretical framework that introduces provider-driver complementarity into an otherwise

[10] To the best of my knowledge, extensive consumer data that allows matching consumers' demand with the different products supplied by a firm is not readily available

Figure 5: Consumption decisions for internet-based services in the U.S.



Note: Survey conducted in the United States from October 26 to November 5, 2018. Respondents aged 18-69. Source: Statista Global Consumer Survey

standard quality ladder model.

3 A theory of provider-driven complementarity

In this section, I describe an endogenous growth model where firms can generate provider-driven complementarities. The model builds upon the quality ladder growth model introduced by Akcigit and Kerr (2018). To clarify and facilitate the exposition and analysis of the effects of provider-driven complementarity, I characterize firms' decisions and equilibrium outcomes in two steps. First, I consider a simplified version of the model where the quality jump obtained after a successful innovation can always offset the effects of provider-driven complementarity, i.e., the latter does not play a role in determining the market leader of each variety. This simplified version shows some key insights into how provider-driven complementarity affects the optimal decisions of firms. I then consider a generalized version of the model where quality and provider-driven complementarity interact and jointly determine each variety's market leader.

3.1 Environment

Time is continuous and infinite. The economy comprises a representative household, a final good producer, and an endogenous measure of (multiproduct) firms that produce a continuum of intermediate goods indexed by $j \in [0,1]$. In what follows, I describe each agent in detail.

3.1.1 Household

There is a representative household with preferences represented by the logarithmic utility function

$$U_t = \int_0^\infty e^{-\rho t} \ln C_t \, \mathrm{d}t,$$

where C_t denotes consumption of the final good of the economy. At any instant, the household is endowed with one unit of labor that supplies inelastically and receives as counterpart the wage rate w_t. The household is also the owner of all firms in the economy, and thus its wealth A_t is simply the aggregate value of all these firms. Denoting by r_t the continuous rate of return on wealth, one can write the household flow budget constraint as

$$\dot{A}_t = w_t + r_t A_t - C_t.$$

The solution to the maximization problem of the household yields the common Euler equation

$$\frac{\dot{C}_t}{C_t} = r_t - \rho. \tag{1}$$

3.1.2 Final goods

There is a unique all-purpose final good that can be used both for consumption and R&D. This final good is produced by aggregating a continuum of intermediate varieties $j \in [0,1]$. At any instant, there is an endogenously determined set $\mathcal{F}$ (of measure $F > 0$) of active incumbent intermediate producers indexed by $i \in \mathcal{F}$, who operate in a monopolistically competitive product market. For each variety $j \in [0,1]$ there exist different vintages characterized by q_{ijt}, the quality of variety j that firm i produces at time t. The quantity of variety j (produced by firm i) demanded in the final good production is denoted by k_{ijt}. The final good production function is given by

$$Y_t = \frac{L_t^\beta}{1-\beta} \left[\int_0^1 \left(\sum_{i \in \mathcal{F}} \left[(m_{it} q_{ijt})^{\frac{1}{\varepsilon-1}} k_{ijt} \right]^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}j \right]^{\frac{\varepsilon(1-\beta)}{\varepsilon-1}}, \tag{2}$$

where $0 < \beta < 1$, $\varepsilon \geqslant 0$ is the elasticity of substitution across different varieties, and $\theta \geqslant 0$ is the elasticity of substitution across different vintages of a given variety. The only non-standard feature in (2) is $m_{it} \geqslant 1$ which denotes the provider-driven complementarity effect derived by demanding goods produced by firm i. Before discussing this novel argument in detail, I impose two parametric restrictions regarding the parameters driving the elasticities of substitution between varieties and different vintages within a variety, θ and ε respectively. First, I assume that the different vintages of a given variety are perfect substitutes once adjusted by quality and provider-driven complementarity, i.e., $\theta = \infty$. Second, for tractability I impose $\varepsilon = \beta^{-1}$.

The idea behind provider-driven complementarity is intuitive and straightforward. Provider-driven complementarity is a mechanism that turns goods into 'complements' when acquired from the same firm. As an example, consider that two firms were able to produce the same good j at the same quality level q_{jt}. In that case, according to (2) in the absence of price differences between both firms, it would be optimal to demand good j from the firm i that gives the highest complementarity. I assume that each firm's level of provider-driven complementarity is only a function of the number of products that it supplies to the market. Formally

$$m_{it} = \gamma^{1-\frac{1}{n_{it}}}, \tag{3}$$

where $\gamma \geqslant 1$ is constant across firms and n_{it} denotes the number of varieties currently provided by firm i. Coming back to the previous example, in the absence of quality differences, (2) will demand good j from the firm with higher n among all firms able to produce j at some given quality q_{jt}. A direct implication of this functional form is that firms' industrial organization, i.e., the equilibrium distribution of the number of goods across firms, matters for the economy's aggregate production.

I assume that intermediate firms compete à la Bertrand. This would naturally lead to the standard limit pricing framework where the firm that can offer the highest quality, adjusted by provider-driven complementarity, relative to its market price – read as the most productive firm – is limited by the second most productive firm.[11] However, to keep the model simple and avoid the case of limit pricing, I follow Akcigit and Kerr (2018) and Garcia-Maza, Hsieh and Klenow (2019) and adopt the following stage-game assumption.[12]

Assumption 1 (Monopoly pricing). *Intermediate firms enter a two-stage price-bidding game when setting prices. In the first stage, every firm pays a fixed fee $\epsilon > 0$ to be able to post a price. In the second stage, prices are revealed.*

Assumption 1 ensures that in the Nash equilibrium of this game, only the firm that can offer the highest provider-adjusted quality to price ratio

$$\frac{m_{it}q_{ijt}}{p_{ijt}} \tag{4}$$

per unit of expenditure in variety j pays the fee and enters the second stage, as any other firm can never recover the fee in the second stage. As that firm is the only firm bidding a price, each variety is produced by precisely one firm, which will always operate as a monopoly. I henceforth refer to such a firm as the market leader of variety j. Consequently, the size of each firm n_{it} ultimately denotes the number of markets in which firm i is the market leader. For simplicity, as in equilibrium, only one firm is active in

[11] I henceforth refer to quality, adjusted by provider-driven complementarity, relative to its market price as provider-adjusted quality to price ratio

[12] In Appendix A.3 I discuss the implications of allowing for limit-pricing in a similar way as in (Peters, 2018).

each market, I re-define each firm's provider-driven complementarity effect by m_{jt} where its dependence on j refers to the market leader of that variety.

Taking the intermediate goods' prices as given, the representative final good producer chooses the quantity of each intermediate good k_{jt}, $j \in [0,1]$ to maximize its profits. Normalizing $P_t^Y = 1$, $\forall t$, dropping time subscripts (to ease notation), and acknowledging that there is only one firm supplying each variety, the problem reads as

$$\begin{aligned} \max_{k_j, L} \quad & Y - \int_0^1 p_j k_j \,\mathrm{d}j - wL \\ \text{s.t.} \quad & \frac{L^\beta}{1-\beta} \int_0^1 [m_j q_j]^\beta k_j^{1-\beta}, \end{aligned}$$

which yields the well-known inverse demand function

$$p_j = L^\beta [m_j q_j]^\beta k_j^{-\beta}, \tag{5}$$

for any variety j.

3.1.3 Intermediate producers

Intermediate producers are characterized by

- the set $\mathcal{J}_i = \{j : j \text{ is owned by firm } i\}$, with cardinality $n \in \mathbb{Z}_+$, the # of varieties in which i is the outstanding market leader,
- the multiset $\mathbf{q}_i = \{q_j : j \in \mathcal{J}_i\} \in \mathbb{R}_+^n$, which collects the quality level of each variety $j \in \mathcal{J}_i$.

Each intermediate variety is produced with the production technology $k_j = \bar{q} l_j$, where

$$\bar{q} = \int_0^1 q_j \,\mathrm{d}j$$

denotes the average quality level of the economy. Under Assumption 1, in equilibrium each intermediate is produced by a single firm – its market leader – which acts as a monopolist. Therefore, each monopolist chooses labor input (and thus quality supplied), its price, and R&D expenditure in order to maximize its profits. I focus first on the intra-temporal labor-quality-price decision of the monopolist. Taking as given (5), each monopolist solves

$$\begin{aligned} \max_{p_j, k_j} \quad & p_j k_j - w l_j \\ \text{s.t.} \quad & k_j = \bar{q} l_j, \\ & p_j = L^\beta [m_j q_j]^\beta k_j^{-\beta}, \end{aligned}$$

which yields the optimal quantity supplied

$$k_j = \left(\frac{\bar{q}(1-\beta)}{w} \right)^{\frac{1}{\beta}} L m_j q_j, \tag{6}$$

at the price

$$p_j = \frac{1}{1-\beta}\frac{w}{\bar{q}}. \tag{7}$$

Note that all monopolists set the same price – a direct consequence of Assumption 1 – but sell different quantities in equilibrium. Intermediate firms' profits in market j are then given by

$$\pi_j = \beta(1-\beta)^{\frac{1-\beta}{\beta}}\left(\frac{\bar{q}}{w}\right)^{\frac{1-\beta}{\beta}} L m_j q_j, \tag{8}$$

for w to be determined.[13] This implies that differences in profits across firms are given by differences in quality, size (number of products), or both.

3.1.4 Innovation

Incumbent firms can invest in R&D activities to improve the quality of their product portfolio by conducting internal R&D. Moreover, incumbent firms can expand their product portfolio by conducting external R&D. Potential entrants can invest in external R&D to enter and become the leaders of a variety. Figure 6 shows the evolution of quality under each type of innovation. The quality of each variety is represented by the vertical bars, and the quality jump obtained after each type of successful innovation is represented by the black bars on top of each variety that is innovated upon. I assume that innovation allows firms to obtain a perpetual patent on a variety-quality pair, i.e., no other firm can supply the same variety at the same level of quality. Given the assumptions made on competition, a successful innovator can become the market leader of a variety and prevent any other firm from supplying the same variety-quality pair to the market. However, the quality of that variety becomes the state-of-the-art or frontier quality upon which other firms can innovate.[14]

Internal innovation I start by focusing on an incumbent firm's decision to improve the quality of one of the goods it supplies to the market. To do that, the firm needs to conduct internal R&D. I assume that internal R&D is directed, i.e., the firm can specifically select in which good it will invest to increase its quality. To create a Poisson flow rate $z_{nj} \geqslant 0$ of improving product j quality, the firm must incur the flow cost

$$C_z(z_{nj}, q_j) = \hat{\chi} z_{nj}^{\hat{\psi}} q_j,$$

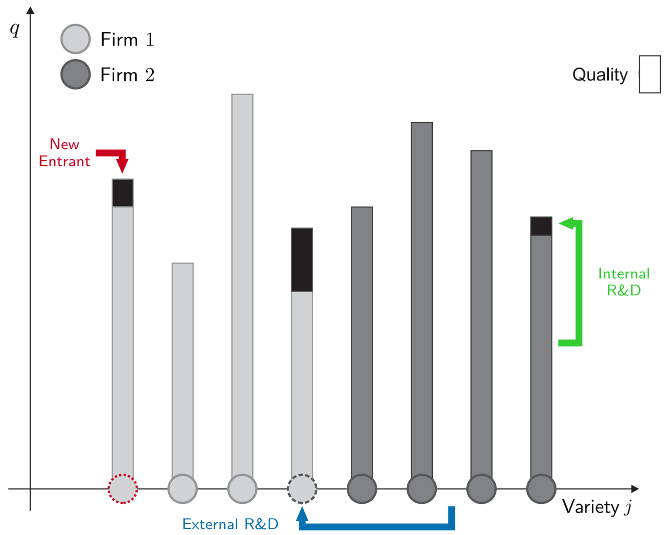
[13] Anticipating a later result, each monopolist per-variety profit is given by

$$\pi_j = \underbrace{\beta^{\beta}(1-\beta)^{2-2\beta} L \left(\frac{\bar{q}}{\bar{Q}}\right)^{1-\beta}}_{\pi} m_j q_j.$$

Profits thus depend on a constant term across firms π, which increases in $\bar{q}$ and decreases in $\bar{Q}$, the average provider-adjusted quality of the economy, to be defined later.

[14] Patents can therefore be considered a protective tool that firms use to pre-empt competitors from entering their product market. As a consequence, by construction, this model exhibits a tight relationship between patents and innovation. However, previous work shows that the relationship between patents and innovation is complex. See for example Argente, Baslandze, Hanley and Moreira (2020).

Figure 6: Innovation types. Cf. (Akcigit and Kerr, 2018, Figure 5).



in terms of the final good of the economy, where $\hat{\chi} > 0$ is a scale parameter and $\hat{\psi} > 1$ implies that the cost function is convex in z_{nj}. If R&D is successful, the firm is able produce good j with incremental quality $q_{jt+\Delta t} = q_{jt}(1 + \lambda_z) \equiv \Lambda_z$, where $\lambda_z > 0$ is assumed to be constant.

External innovation An incumbent firm can also perform external R&D to improve the quality of a product outside its quality portfolio, and become the market leader of that good. I assume that external R&D is undirected. An incumbent firm with product portfolio size $n \geqslant 1$ chooses the Poisson flow rate $X_n \geqslant 0$ with which it improves the quality of an already existing product not currently owned. For a given flow cost in external R&D C_x, the flow rate of external innovations is given by the innovation function

$$X_n = \chi(C_x/\bar{q})^{\psi} n^{\sigma},$$

which delivers the R&D flow cost function

$$C_x(x_n, n, \bar{q}) = \tilde{\chi} x_n^{\tilde{\psi}} n^{\tilde{\sigma}} \bar{q}, \tag{9}$$

in terms of the final good, where $x_n = X_n/n$ is the per-product flow rate of external R&D and $\tilde{\chi} = \chi^{-\frac{1}{\psi}} > 0$ is a scale parameter, $\tilde{\psi} = 1/\psi$ and $\tilde{\sigma} = (1 - \sigma)/\psi$. The relationship between $\psi > 0$ and $\sigma > 0$ determines the returns to scale of the innovation function. In particular, if $\psi + \sigma = 1$ it exhibits constant returns to scale as in (Klette and Kortum, 2004), while if $\psi + \sigma =< 1$ it exhibits decreasing returns to scale as in (Akcigit and Kerr, 2018). If R&D is successful, a firm can produce any randomly drawn product (not previously owned) with incremental quality $q_{jt+\Delta t} = q_{jt}(1 + \tilde{\lambda}_x) \equiv q_{jt}\Lambda_x$, with $\tilde{\lambda}_x > 0$ drawn from an exponential distribution with parameter λ_x^{-1}. Unlike most previous work in the literature, in this environment a firm that innovates and improves the quality of a product does not necessarily become its new producer.[15] The innovator will only become the new market leader if it can offer the highest provider-adjusted quality to price ratio.

[15]For another recent example, see De Ridder (2020).

As a consequence, firms with larger product portfolios, and firms receiving higher quality improvements, are more likely to become market leaders.

In what follows, I describe how quality improvements interact with provider-driven complementarity in determining the market leader of each variety. Let I denote a firm obtaining a quality improvement in some product line, and let L denote the current market leader in that product line. Moreover, let m_I and m_L denote the provider-driven complementarity effect generated by the innovating firm and the current market leader, respectively. I assume that the economy follows the behavioral through which the innovating firm I (which could either be a brand-new entrant or an incumbent with outstanding market leadership in at least one variety) will become the new producer of a variety if it can price-out the incumbent by offering a higher provider-adjusted quality to price ratio, i.e., if

$$\frac{m_I}{m_L}\Lambda_x > 1.$$

Equivalently,

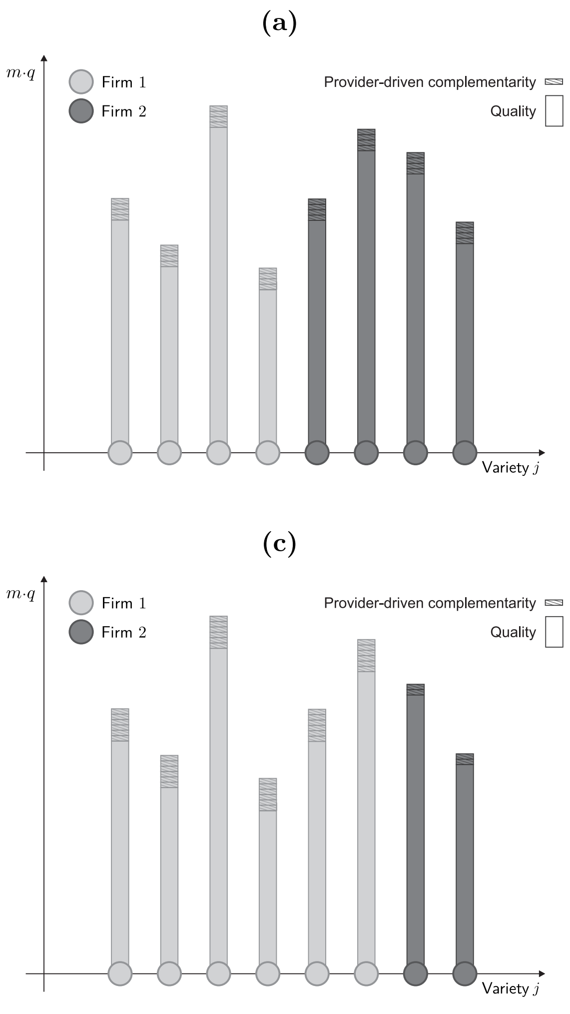
$$\Lambda_x \geqslant \gamma^{\frac{1}{n_I+1}-\frac{1}{n_L}} \equiv \gamma^{\Delta(n_I+1,n_L)}. \tag{10}$$

This expression implies that the quality jump needed to price-out the incumbent must be sufficiently large to offset the provider-driven complementarity gap. Note that the left-hand side is always bigger than one, while the right-hand side may be bigger, equal, or smaller than 1 and is a function of the number of products currently produced by the incumbent and the innovator. Consequently, for a given quality jump Λ_x, firms already selling many different varieties find it easier to become market leaders than smaller firms. Put it differently; for a given size of an incumbent market leader, smaller firms need bigger quality jumps than bigger firms to become market leaders.

To provide further intuition, in Figure 7 I consider two possible situations where, without loss of generality, I consider an economy with two firms competing to supply eight different varieties. First, panels 7a and 7b depict a situation where both firms are initially supplying the same number of goods. Panel 7a shows that, under provider-driven complementarity, the provider-adjusted quality enjoyed by the consumer is composed of two parts: the variety-specific quality, represented as in the Baseline framework with a solid bar, and the provider-driven complementarity, represented by the four hatched bars for each variety. Now, suppose that Firm 2 is successful in conducting external R&D, being now able to produce one of Firm 1 varieties with higher quality. Suppose the quality jump over Firm 1 in that variety is represented by the solid black bar in panel Panel 7b. Note that, as both firms have the same initial size, the quality jump is not key to determine the new market leader of that variety, because the consumer has a double incentive to switch providers for that variety: Firm 2 quality is higher, and it increases the provider-driven complementarity effect obtained by the consumer not only in that variety but also in all the remaining varieties supplied by Firm 2. This increase in provider-driven complementarity of Firm 2 is represented by the white hatched bar. Note also that the consumer enjoys less provider-driven complementarity in all the varieties still supplied by Firm 1 so that she would not be willing to pay the same price as before. This is a novel price-effect given by provider-driven complementarity. Firm 1 will necessarily re-optimize prices to keep maximizing its profits.

Panels 7c and 7d depict a situation where both firms are initially supplying a different number of varieties. As before, Panel 7c represents an initial situation where Firm 1 is

Figure 7: External innovation under provider-driven complementarity.



supplying six varieties while Firm 2 is supplying the remaining two. Again, suppose that Firm 2 is successful in conducting external R&D, being now able to produce one of Firm 1 varieties with higher quality. Now the interaction between the quality jump and each firm's size becomes key to determine the new market leader. The black bar in Panel 7d represents the quality jump that would make the consumer indifferent between sticking with the lower quality version supplied by Firm 1 or switching to the brand-new better quality version offered by Firm 2. In other words, any quality jump bigger than the one represented in Panel 7d would make Firm 2 become the new market leader, while any quality jump smaller would allow Firm 1 to keep its market leadership.

As the quality increase is stochastic, the probability of condition (10) being satisfied is characterized by[16]

$$\Pr\left(\Lambda_x \geqslant \gamma^{\Delta(n_I+1,n_L)}\right) = \exp\left(-\frac{1}{\lambda_x}\left[\gamma^{\Delta(n_I+1,n_L)} - 1\right]\right). \tag{11}$$

[16]For an exponential distribution with parameter λ, $\Pr(X < x) = F_X(x;\lambda) = 1 - e^{-\lambda x}$, and thus $\Pr(X > x) = e^{-\lambda x}$.

Note that the fact that firms endogenously increase or decrease their size over time makes condition (10) dynamic. I assume that once a firm has been priced out of a product market, it is no longer a competitor in that product market.[17] This assumption avoids the possibility of losing the market leadership of a product even if no other firm improves the quality of that product, but simply because a second firm that produces this product has improved the quality of a different product becomes its market leader. I assume that this is also the case if an innovation is not successful in pricing out an incumbent. In other words, even if a firm can improve the quality of a product, that quality improvement is lost if it is not big enough to become the market leader. The underlying idea implies that: 1. Once a firm has been priced out of a market, it dismantles the production capacity for a product and can no longer supply it; 2. If a product is not sufficiently good to be demanded by consumers, a firm has to start a new R&D project from scratch (which in this context is the quality of the leader) and improve quality upon it.

Finally, I assume that incumbent firms incur adjustment costs when their size changes due to external innovation. This adjustment cost is assumed to be an increasing function of the per-product external innovation rate x_n, the firm's aggregate quality, and its size. In other words, increasing the portfolio of a firm is costlier for firms with high-quality products. Formally, firms pay

$$C_a(x_n, n, \mathbf{q}_i) = x_n \Omega_n \sum_{q_j \in \mathbf{q}_i} q_j,$$

with Ω_n (weakly) decreasing in n. This assumption serves two purposes. First, it allows obtaining closed-form solutions to the value function of the firm. Second, it ensures that there exists a balanced growth path.[18]

Entry As is standard in the literature, a mass of entrants invest in R&D in order to become intermediate producers of a variety. Entrants choose an innovation flow rate $x_e > 0$ with an R&D cost

$$C_e(x_e, \bar{q}) = \nu x_e \bar{q},$$

in terms of the final good, where $\nu > 0$.

Creative destruction Successful innovation by entrants and incumbent firms can cause other incumbent firms to lose production of the goods that were innovated upon. The rate at which this happens is the creative destruction rate, τ_n. As the probability of obtaining a successful innovation depends on the interplay between quality improvement and provider-driven complementarity, which is ultimately a function of size, the rates of creative destruction suffered by firms of different sizes will differ. These rates are

[17] Recall that the incumbent is already behaving like a monopolist due to the two-stage price-bidding game.

[18] Without this assumption, the incentives to conduct external R&D not only depend on the size of a firm but also on its specific quality portfolio. That implies that all else equal, big high-quality firms have more incentives to innovate than big small-quality firms. The adjustment cost breaks the dependence of R&D on the quality portfolio of the firm. Specifically, it ensures that the incentive to conduct external R&D depends on the profits associated with becoming a market leader of a new product incorporated into the firms' product portfolio, as is standard in this literature.

endogenous and are determined from the optimal R&D decisions of firms. The per-product rate of creative destruction suffered by a firm with $n > 0$ products is

$$\tau_n = x_e \Pr\left(\Lambda_x \geqslant \gamma^{\Delta(1,n)}\right) + \sum_{s=1}^{\infty} F\mu_s X_s \Pr\left(\Lambda_x \geqslant \gamma^{\Delta(s+1,n)}\right).$$

The first term in this expression captures the creative destruction rate suffered by a firm of size n by new entrants in the economy. This is given by the entrant innovation intensity times the probability of the quality jump being sufficiently big to offset the provider-driven complementarity effect of a size n firm. Similarly, the summation captures the creative destruction rate suffered as a consequence of the innovation activity of already existing firms producing $n \geqslant 1$ products, where μ_s is the invariant share of firms producing s products (to be determined in equilibrium).

Finally, an incumbent firm exits the economy if it loses the market leadership of its only product.

3.1.5 Aggregate variables

To conclude the exposition of the environment and before turning to the characterization of firms' dynamic decisions, it remains to derive some key aggregate variables. I start by deriving the equilibrium wage. Substituting the optimal intermediate quantity supplied (6) in the labor demand optimality condition of the final good producer's problem yields

$$w = \beta \frac{L^{\beta-1}}{1-\beta} \int_0^1 [m_j q_j]^{\beta} \left(\frac{\bar{q}(1-\beta)}{w}\right)^{\frac{1-\beta}{\beta}} L^{1-\beta} [m_j q_j]^{1-\beta} \, \mathrm{d}j,$$

which can be expressed as

$$w = \beta^{\beta}(1-\beta)^{1-2\beta} \bar{q}^{1-\beta} \bar{Q}^{\beta}, \tag{12}$$

where

$$\bar{Q} = \int_0^1 m_j q_j \, \mathrm{d}j, \tag{13}$$

denotes the average provider-adjusted quality of the economy.[19]

Aggregate output can be obtained by substituting the optimal intermediate quantity supplied (6) and the equilibrium wage (12) into the final good production function (2), obtaining

$$Y = \beta^{\beta-1}(1-\beta)^{1-2\beta} L \bar{q}^{1-\beta} \bar{Q}^{\beta}. \tag{14}$$

The share of workers in the production of the final good can be obtained from the labor market clearing condition

$$L + L^k = 1,$$

where

$$L^k = \int_0^1 l_j \, \mathrm{d}j = \int_0^1 \frac{k_j}{\bar{q}} \, \mathrm{d}j.$$

[19]Note that $\bar{Q} \geqslant \bar{q}$ with equality if an only if $m_j = 1$, $\forall j$.

Substituting the optimal intermediate quantity supplied (6) and the equilibrium wage (12) in the previous expression yields

$$L^k = \frac{(1-\beta)^2}{\beta} L.$$

Therefore, from the labor market clearing condition one obtains that employment in the final good sector represents a constant share of total employment given by

$$L = \frac{\beta}{\beta + (1-\beta)^2}. \tag{15}$$

Along a balanced growth path, aggregate variables will grow a constant rate g. As L is constant, from (14) it is clear that output will grow at the same rate as $\bar{q}^{1-\beta}\bar{Q}^{\beta}$. The rest of this section characterizes the growth rate of these two components. First, I derive the growth rate of the average quality $\bar{q}$. Recovering the time subscript, and defining

$$z_t = \sum_{n=1}^{\infty} F\mu_n n z_n,$$
$$\tau_t = \sum_{n=1}^{\infty} F\mu_n n \tau_n,$$

the evolution of $\bar{q}_t$ during any interval $[t, t+\Delta t]$ can be expressed as

$$\bar{q}_{t+\Delta t} = \bar{q}_t(1+\lambda_x)\tau_t\Delta t + \bar{q}_t(1+\lambda_z)z_t\Delta t + \bar{q}_t(1-\tau_t - z_t)\Delta t + o(\Delta t),$$

i.e., in expectation the average quality of the economy can either increase by the factor λ_x due to creative destruction τ_t (which occurs for a fraction τ_t of products), or it can increase by the fixed factor λ_z due to internal innovation z_t (which occurs for a fraction z_t of products), or may remain unchanged (which occurs for a fraction $1-\tau_t\Delta t - z_t\Delta t$ of products). Re-arranging the previous expression and taking limits as $\Delta t \to 0$ yields

$$g \equiv \lim_{\Delta t \to 0} \frac{\bar{q}_{t+\Delta t} - \bar{q}_t}{\Delta t} = \lambda_z z_t + \lambda_x \tau_t. \tag{16}$$

Finally, the next proposition establishes the growth rate of the average provider-adjusted quality $\bar{Q}$.

Proposition 1. *Along the balanced growth path*

$$\frac{\partial \ln \bar{Q}}{\partial t} = \frac{\partial \ln \bar{q}}{\partial t} = g.$$

Proof. Appendix A.2.1. ■

The previous proposition anticipates an important equilibrium result. Along the balanced growth path, the equilibrium firm size distribution will be constant. Consequently, along the balanced growth path, provider-adjusted quality grows at the same rate as average quality, and so does aggregate output.

3.2 Equilibrium

In this subsection, I analyze the dynamic R&D decisions of firms. To facilitate the exposition, I divide the Subsection into two parts. The first part considers a simplified version of the model that restricts the analysis to external innovations with quality improvements that always offset provider-driven complementarity. This allows introducing notation, simplifies the characterization of firms' decisions and equilibrium outcomes, and helps obtain some key insights regarding how provider-driven complementarity affects firms' optimal decisions. The second part extends the analysis to the more general case where quality and provider-driven complementarity interact and jointly determine each variety's market leader.

3.2.1 Simplified framework

I start the analysis of the dynamic R&D decisions of firms under provider-driven complementarity by restricting to a simple case where external innovations improve the quality of a variety so that the effect of provider-driven complementarity is always offset. The following assumption ensures this result.

Assumption 2. *Upon external innovation, quality improves by a fixed step λ_x s.t. $\Lambda_x > \gamma$.*

The value γ is an upper bound for the provider-driven complementarity effect. Therefore Assumption 2 implies that after successful external R&D, a firm can produce any randomly drawn product (not previously owned) with incremental quality $q_{jt+\Delta t} = q_{jt}(1+\lambda_x) \equiv q_{jt}\Lambda_x$, with $\lambda_x > 0$ being a constant. This is the only modification concerning the previous Subsectionenvironment but is key for the results. The next proposition shows that under Assumption 2, provider-driven complementarity is not relevant in determining the market leader of a variety.

Proposition 2. *Under Assumption 2, a firm that is successful in conducting external R&D and improves the quality of a product, becomes its new producer independently of the number of products it supplies to the market.*

Proof. It is straightforward to show that under Assumption 2, the firm providing the highest quality of a variety is always able to offer the highest provider-adjusted quality to price ratio, as the incremental quality jump obtained through external R&D (λ_x) is always bigger than the maximum possible effect of provider-driven complementarity (γ). ■

Proposition 2 implies that the probability of condition (10) being satisfied – obtaining a successful innovation that allows to replace a incumbent – is always equal to 1, and specifically does not depend on the size of the innovator or the incumbent. As a consequence the rate of creative destruction suffered by a firm with $n \geqslant 1$ is now given by

$$\tau = x_e + \sum_{s=1}^{\infty} F\mu_s s x_s. \tag{17}$$

In order words, the rate of creative destruction is constant across firms of different sizes. This implies that any firm is equally likely to lose a product through creative destruction.

Taking as given the values of r, g, and τ, the value functions of firms determine their optimal dynamic R&D decisions. I start with the value function of an incumbent firm.

Consider a firm $i \in \mathcal{F}$ being the market leader of $n \geqslant 1$ varieties. Such a firm chooses the per-product external, x_n, and internal, z_{nj}, innovation rates in order to maximize[20]

$$
\begin{aligned}
rV_n(\mathbf{q}_i) = \max_{x_n,\ \{z_{nj}\}_{j=1}^{n}} \Bigg\{ & \sum_{q_j \in \mathbf{q}_i} \Big[\pi \gamma^{1-n^{-1}} q_j + z_{nj} \big(V_n(\mathbf{q}_i \backslash_{-} \{q_j\} \cup_{+} \{q_j \Lambda_z\}) - V_n(\mathbf{q}_i) \big) - \hat{\chi} z_{nj}^{\hat{\psi}} q_j \\
& + \tau \big(V_{n-1}(\mathbf{q}_i \backslash_{-} \{q_j\}) - V_n(\mathbf{q}_i) \big) \Big] \\
& + n x_n \Big(\mathbb{E}_{h,\lambda_x} \left[V_{n+1}(\mathbf{q}_i \cup_{+} \{q_h \Lambda_x\}) \right] - V_n(\mathbf{q}_i) - \Omega_n \sum_{q_j \in \mathbf{q}_i} q_j \Big) - \tilde{\chi} x_n^{\tilde{\psi}} n^{\tilde{\sigma}} \bar{q} \Bigg\} \\
& + \dot{V}_n(\mathbf{q}_i)
\end{aligned}
$$

The first line on the right-hand side collects the profits obtained per each variety in which the firm is the market leader and the change in value if internal innovation is successful for each of those varieties. The second line captures the change in value experienced from losing each of the varieties through creative destruction. The third line captures the increase in value due to external innovation. If the firm's R&D is successful, it improves the quality of any product h outside its quality portfolio and becomes the new market leader of that variety. Given that R&D is undirected, the new product's quality is unknown and is captured by the expected value $\mathbb{E}_{h,\lambda_x}$, which is an expectation over quality level and innovation step. Finally, the last line captures the change in firm value due to the economy's growth along its balanced growth path. The following proposition characterizes the closed-form solution of the value function.

Proposition 3. *There exists an equilibrium with adjustment cost $\Omega_n = A_{n+1} - A_n$, and positive entry, where for $n \geqslant 1$ an incumbent's value function has the form*

$$
V_n(\mathbf{q}_i, \bar{q}) = A_n \sum_{q_j \in \mathbf{q}_i} q_j + B_n \bar{q}, \tag{18}
$$

and the coefficients A_n, B_n and Ω_n satisfy the recursions

$$
(r + \tau n) A_n = \pi \gamma^{1-n^{-1}} + \hat{\chi}(\hat{\psi} - 1) \left(\frac{A_n \lambda_z}{\hat{\chi} \hat{\psi}} \right)^{\frac{\hat{\psi}}{\hat{\psi}-1}} + \tau (n-1) A_{n-1}, \tag{19}
$$

$$
B_{n+1} = B_n - A_{n+1} \Lambda_x + \tilde{\psi} \tilde{\chi}^{\frac{1}{\tilde{\psi}}} n^{\frac{\tilde{\sigma}-\tilde{\psi}}{\tilde{\psi}}} \left(\frac{(\rho + n\tau) B_n - n\tau B_{n-1}}{\tilde{\psi} - 1} \right)^{\frac{\tilde{\psi}-1}{\tilde{\psi}}}, \tag{20}
$$

with $A_0 = B_0 = 0$. Moreover, the per-product external and internal innovation rates are given by

$$
x_n = \left(\frac{A_{n+1} \Lambda_x + B_{n+1} - B_n}{\tilde{\psi} \tilde{\chi} n^{\tilde{\sigma}-1}} \right)^{\frac{1}{\tilde{\psi}-1}}, \tag{21}
$$

$$
z_{nj} = z_n = \left(\frac{A_n \lambda_z}{\hat{\chi} \hat{\psi}} \right)^{\frac{1}{\hat{\psi}-1}}. \tag{22}
$$

Proof. Appendix A.2.3. ■

[20] A detailed derivation can be found in Appendix A.2.2.

The value function (18) consists of two parts. The first part A_n captures the expected discounted stream of profits obtained by being the variety's market leader. Its dependence on size stems from the profit function (8). Besides, it also incorporates the value of improving the quality of each variety through internal R&D. The second part B_n relates to the value of extending the product portfolio of the firm through external R&D. The equilibrium per-product internal innovation rate depends on size as, all else equal, bigger firms generate more provider-driven complementarity, yielding a higher marginal gain from improving the quality of each one of their varieties. However, it is independent of quality as both the profit and internal cost function are linear in quality. Size also affects the equilibrium per-product external innovation rate for two reasons. First, it directly affects the production function of external innovations but, most importantly, affects the firm's provider-driven complementarity. As a firm grows, it generates more complementarity, which affects its incentives to conduct external R&D.

It remains to characterize the R&D decision of an outside entrepreneur (potential entrant). Denote by V_0 the value of a firm with size 0, i.e., a firm with an empty quality portfolio. Taking as given the values of r, g and τ, an outside entrepreneur chooses the entry rate x_e in order to maximize

$$rV_0 - \dot{V}_0 = \max_{x_e} \left\{ x_e \left(\mathbb{E}_{h,\lambda_x} \left[V_1(\{q_h \Lambda_x\}) - V_0 \right] \right) - \nu x_e \bar{q} \right\}, \tag{23}$$

where $V_1(\{q_h \Lambda_x\})$ denotes the value of a firm that owns a single product line with quality $q_h \Lambda_x$ and $\dot{V}_0 \equiv \partial V_0 / \partial t$ captures the change in the value of being an outside entrepreneur due to the growth of the economy along its balanced growth path. The expected value $\mathbb{E}_{h,\lambda_x}$ is an expectation over quality level q_h and innovation step $\tilde{\lambda}_x$. Therefore, an outside entrepreneur's value is determined by the expected value from obtaining an external innovation and replacing an incumbent.

To conclude the equilibrium's characterization, I still need to derive the invariant distribution of the number of products across firms. Recall that, as laid out before, μ_n denotes the share of firms producing n goods, and it satisfies $\sum_{n=1}^{\infty} \mu_n = 1$. The invariant distribution depends on the flow equations

n	Inflows	Outflows
0	$F\mu_1\tau$	x_e
1	$F\mu_2 2\tau + x_e$	$F\mu_1(\tau + x_1)$
$\vdots$	$\vdots$	$\vdots$
n	$F\mu_{n+1}(n+1)\tau + F\mu_{n-1}(n-1)x_{n-1}$	$F\mu_n n(\tau + x_n)$

The first row shows the inflows and outflows into outside entrepreneurs. An inflow happens whenever a firm with one product loses it through creative destruction, either to another outside entrepreneur or an incumbent firm. Outflows happen when an outside entrepreneur obtains a successful innovation and becomes an incumbent with one single product. The second row shows the inflows and outflows of firms with market leadership in one variety. Inflows occur when outside entrepreneurs obtain a successful innovation and whenever a firm with two products loses one through creative destruction. Outflows happen if a firm loses its only product and becomes an outside entrepreneur or when a firm with one product obtains a successful innovation and increases its portfolio of varieties. The last row is simply a generalization for firms with any number of varieties $n > 1$. Finally, the following proposition derives a closed-form solution of the invariant product number distribution.

Proposition 4. *For $n \geqslant 1$, the invariant distribution μ_n is given by*

$$\mu_n = \frac{1}{n} \frac{x_e}{F x_n} \prod_{s=1}^{n} \frac{x_s}{\tau}. \tag{24}$$

Proof. Appendix A.2.4. ■

Before introducing the definition of the balanced growth path equilibrium of this economy, which will conclude this subsection, it is helpful to obtain an expression for aggregate R&D expenditure R. This is given by the sum of external R&D expenditure of incumbents x_n (including adjustment costs) and potential entrants x_e, and internal R&D expenditure of incumbents z_n. Therefore, R can be obtained as

$$R = \sum_{i=1}^{n} F\mu_n \left[x_n^{\tilde{\psi}} n^{\tilde{\sigma}} \bar{q} + \sum_{j=1}^{n} \left(n\Omega_n + \hat{\chi} z_n^{\hat{\psi}} \right) q_j \right] + \nu x_e \bar{q}. \tag{25}$$

Next, a balanced growth path equilibrium and its properties are defined.

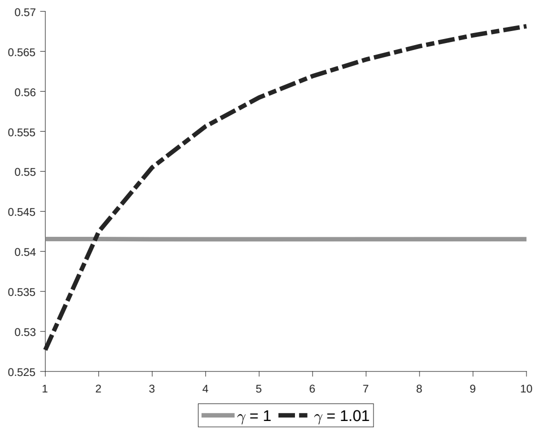
Definition 3.1. *For every t, $j \in [0,1]$, n, $\bar{q}$ and q_j, a balanced growth path equilibrium consists of allocations $\{k_j, l_j, \pi_j, x_n, z_n, x_e\}$, provider-driven complementarity $\{m_j\}$, aggregate variables $\{Y, C, R, A, L, L^x, F, \bar{Q}\}$, value function coefficients $\{A_n, B_n\}$, distribution of number of products $\{\mu_n\}$, rates $\{\tau, g\}$ and prices $\{p, w, r\}$ such that*

1. *$\{k_j, l_j, \pi_j, p\}$ are the solution to the intra-temporal labor-quality-price decision of intermediate firms,*
2. *per-product external $\{x_n\}$ and internal $\{z_n\}$ flow rates satisfy (21) and (22),*
3. *$\{w\}$ satisfies (12),*
4. *share of workers producing the final good L and intermediate goods L^k satisfy (15) and $L^k = 1 - L$,*
5. *creative destruction τ satisfies (17),*
6. *entry flow rate x_e and measure of incumbents F solve (23), and satisfy the free-entry condition $V_0 = 0$,*
7. *distribution of number of products satisfies (24),*
8. *given μ_n, provider-driven complementarity $\{m_j\}$ and average provider-adjusted quality $\bar{Q}$ satisfy (3) and (13),*
9. *aggregate R&D, R, satisfies (25), aggregate output Y satisfies (2) and aggregate consumption satisfies the market clearing condition $C = Y - R$,*
10. *the growth rate satisfies (16),*
11. *the value function coefficients $\{A_n, B_n\}$ satisfy (19) and (20),*
12. *the interest rate satisfies the Euler equation (1).*

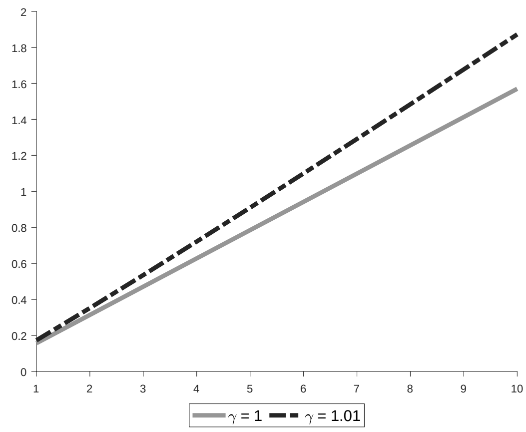
The effects of provider-driven complementarity In this section, I explore the equilibrium implications of provider-driven complementarity. Figure 8 compares an economy without provider-driven-complementarity ($\gamma = 1$) with an economy with provider-driven-complementarities ($\gamma = 1.01$). Otherwise, the parametrization is the same for both economies and is chosen for illustrative purposes.

Figure 8: The effects of provider-driven complementarity

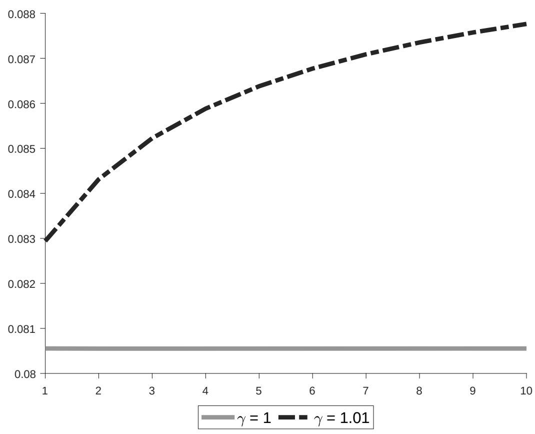
(a) Value function coefficients A_n



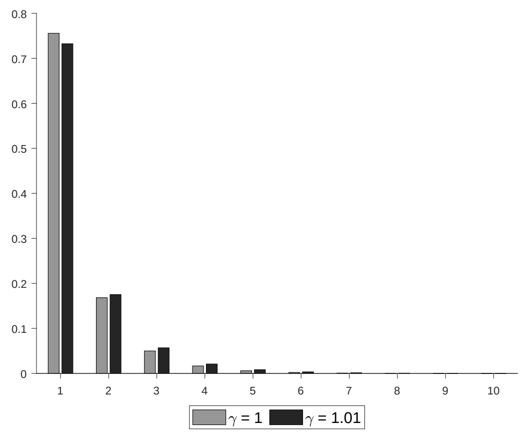
(b) Value function coefficients B_n



(c) Per-product external innovation rate



(d) # products distribution



Under provider-driven complementarity, the per-variety profit function (8) shows that the profits of a firm increase in the number of products supplied to the market. As a consequence, the first term A_n in the value function 18 also does, as it captures the discounted stream of profits obtained by a market leader. Panel 8a shows this result. Note that the value of being an incumbent with one product if firms generate provider-driven complementarity is lower than there is no provider-driven complementarity. This happens due to the different rates of creative destruction between both economies. As the value of being an incumbent firm increases in firm size, the franchise value of expanding into more product lines, captured by the second term in the value function 18, is slightly convex as Panel 8b shows. Hence, the per-product external innovation rates increase in firm size, leading to a higher creative destruction rate in equilibrium. This increase implies that incumbents expect to be replaced faster, reducing their discounted stream of benefits. This higher equilibrium creative destruction rate is compensated for by the direct effect of provider-driven complementarity on the profit function for firms that are market leaders in more than one variety. This is not the case for firms with one variety that do not generate provider-driven complementarity.

The effects of provider-driven complementarities on the R&D decisions of firms lead to significant changes in the equilibrium invariant firm size distribution as shown in Panel 8d. In particular, the increase in the creative destruction rate leads to a decline in new firms' entry rate as, upon entry, the discounted stream of benefits of an entrant is lower. Consequently, the measure of incumbent firms is also reduced and, most importantly, incumbent firms become bigger.

3.2.2 Baseline framework

The previous discussion highlights that provider-driven complementarity shapes R&D decisions of firms even in a simple model where complementarity does not interact with quality in determining the market leader of each variety. Recall that in the simplified framework both the probability of obtaining a successful innovation and the creative destruction rates experienced by firms are independent of their size (number of varieties in which they are market leaders). This section considers a generalized version of the model where condition 10 – which determines the market leader of each variety – depends both on the quality jump obtained by an innovator, and on the relationship between its size and that of the incumbent.

Taking as given the values of r, g, $\{\tau_n\}$, and $\{\mu_n\}$, an incumbent firm $i \in \mathcal{F}$ chooses the per-product external, x_n, and internal, z_{nj}, innovation rates in order to maximize[21]

$$\begin{aligned} rV_n(\mathbf{q}_i) = \max_{x_n, \{z_{nj}\}_{j=1}^n} \Bigg\{ & \sum_{q_j \in \mathbf{q}_i} \Big[\pi\gamma^{1-n^{-1}} q_j + \tau_n \big[V_{n-1}(\mathbf{q}_i \backslash_- \{q_j\}) - V_n(\mathbf{q}_i) \big] \\ & + z_{nj} \big[V_n(\mathbf{q}_i \backslash_- \{q_j\} \cup_+ \{q_j \Lambda_z\}) - V_n(\mathbf{q}_i) \big] - \hat{\chi} z_{nj}^{\hat{\psi}} q_j \Big] \\ & + n x_n \mathbb{E}_{h,\lambda_x} \left[\mathbf{1}_{\{\Lambda_x \geqslant \gamma^{\Delta(n+1,j)}\}} \left[V_{n+1}(\mathbf{q}_i \cup_+ \{q_h \Lambda_x\}) - V_n(\mathbf{q}_i) - \Omega_n \sum_{q_j \in \mathbf{q}_i} q_j \right] \right] \\ & - \tilde{\chi} x_n^{\tilde{\psi}} n^{\tilde{\sigma}} \bar{q} \Bigg\} + \dot{V}_n(\mathbf{q}_i). \end{aligned}$$

There is a key difference with respect to the simplified framework. Now, to become the new producer of a variety, the innovation step has to be sufficiently big so that it can offset the difference in provider-driven complementarity offered by the incumbent and the innovator. This is captured by the indicator function inside the expectation operator $\mathbb{E}_{h,\lambda_x}$, which now depends crucially on the equilibrium distribution of the numbers of products across firms. This implies that to form this expectation correctly, each firm needs to know the firms' equilibrium distribution across products. As a consequence, this model is computationally more intensive, although it remains tractable. The following proposition characterizes the closed-form solution of the value function, closely resembling that of the simplified framework.

[21] A detailed derivation can be found in Appendix A.2.2.

Proposition 5. *There exists an equilibrium with adjustment cost $\Omega_n = A_{n+1} - A_n$, and positive entry, where for $n \geqslant 1$ an incumbent's value function has the form (18), and the coefficients A_n, B_n and Ω_n satisfy the recursions*

$$(r + \tau_n n)A_n - \tau_n(n-1)A_{n-1} = \pi\gamma^{1-n^{-1}} + \hat{\chi}(\hat{\psi} - 1)\left(\frac{A_n\lambda_z}{\hat{\chi}\hat{\psi}}\right)^{\frac{\hat{\psi}}{\hat{\psi}-1}},$$

$$\frac{\mathbb{E}_{h,\lambda_x}\left[\mathbf{1}_{\{\Lambda_x \geqslant \gamma^{\Delta(n+1,j)}\}}\left(A_{n+1}\Lambda_x + B_{n+1} - B_n\right)\right]}{\tilde{\psi}\tilde{\chi}^{\frac{1}{\tilde{\psi}}} n^{\frac{\tilde{\sigma}-\tilde{\psi}}{\tilde{\psi}}}} = \left(\frac{(\rho + n\tau)B_n - n\tau B_{n-1}}{\tilde{\psi} - 1}\right)^{\frac{\tilde{\psi}-1}{\tilde{\psi}}},$$

with $A_0 = B_0 = 0$, and per-product R&D intensities given by

$$x_n = \left(\frac{\mathbb{E}_j\left[\mathbf{1}_{\{\Lambda_x \geqslant \gamma^{\Delta(n+1,j)}\}}\left(A_{n+1}\Lambda_x + B_{n+1} - B_n\right)\right]}{\tilde{\psi}\tilde{\chi}n^{\tilde{\sigma}-1}}\right)^{\frac{1}{\tilde{\psi}-1}}, \tag{26}$$

$$z_{nj} = z_n = \left(\frac{A_n\lambda_z}{\hat{\chi}\hat{\psi}}\right)^{\frac{1}{\hat{\psi}-1}}. \tag{27}$$

Proof. The proof follows closely that of Proposition 3. ■

The value function in the baseline framework also consists of two parts. Note that the key difference with the simplified framework is that the probability of obtaining a successful innovation is now endogenous and appears both explicitly and implicitly through its effect in the creative destruction rate τ_n. The term A_n, in addition to its size dependence due to the shape of the profit function, now additionally depends on size due to the effect of provider-driven complementarity on creative destruction. This is a direct consequence of the fact that all else equal, firms with market leadership over a higher number of goods are less likely to lose their leadership through creative destruction. The probability of obtaining a successful innovation also appears in the recursion for B_n, which relates to the value of extending the product portfolio of the firm through external R&D, and in the per-product external innovation rates x_n. As firms increase their quality portfolio, the probability of obtaining successful innovations increases, but at a declining rate. In other words, the marginal probability increase is declining in size (asymptotically converging to 0 as the probability approaches 1).

As in the simplified framework, the value V_0 of being an outside entrepreneur is the expected value from entering and successfully replacing a incumbent. This value satisfies

$$rV_0 - \dot{V}_0 = \max_{x_e}\left\{x_e\mathbb{E}_{h,\lambda_x}\left[\mathbf{1}_{\{\Lambda_x \geqslant \gamma^{\Delta(1,j)}\}}[V_1(\{q_h\Lambda_x\}) - V_0]\right] - \nu x_e\bar{q}\right\}.$$

Similarly to the case of an incumbent, to become the new producer, the innovation step has to be sufficiently big to offset the difference in provider-driven complementarity offered by the incumbent and the innovator. The indicator function again captures this.

The invariant measure of firms producing n goods, μ_n, needs to be re-defined to incorporate the probability of obtaining a successful innovation. The invariant distribution now depends on the flow equations

n	Inflows	Outflows
0	$F\mu_1\tau_1$	$x_e\mathbb{E}_{h,\lambda_x}\left[\mathbf{1}_{\{\Lambda_x \geqslant \gamma^{\Delta(1,j)}\}}\right]$
1	$F\mu_2 2\tau_2 + x_e\mathbb{E}_{h,\lambda_x}\left[\mathbf{1}_{\{\Lambda_x \geqslant \gamma^{\Delta(1,j)}\}}\right]$	$F\mu_1\tau_1 + F\mu_1 x_1\mathbb{E}_{h,\lambda_x}\left[\mathbf{1}_{\{\Lambda_x \geqslant \gamma^{\Delta(2,j)}\}}\right]$
$\vdots$	$\vdots$	$\vdots$
n	$F\mu_{n+1}(n+1)\tau_{n+1}$	$F\mu_n n\tau_n$
	$+F\mu_{n-1}(n-1)x_{n-1}\mathbb{E}_{h,\lambda_x}\left[\mathbf{1}_{\{\Lambda_x \geqslant \gamma^{\Delta(n,j)}\}}\right]$	$+F\mu_n x_n\mathbb{E}_{h,\lambda_x}\left[\mathbf{1}_{\{\Lambda_x \geqslant \gamma^{\Delta(n+1,j)}\}}\right]$

The following proposition derives a closed-form solution of the invariant product number distribution.

Proposition 6. *For $n \geqslant 1$, the invariant distribution μ_n is given by*

$$\mu_n = \frac{1}{n}\frac{x_e}{Fx_n}\prod_{s=1}^{n}\frac{x_s}{\tau_s}\mathbb{E}_{h,\lambda_x}\left[\mathbf{1}_{\{\Lambda_x \geqslant \gamma^{\Delta(s,j)}\}}\right].$$

Proof. The proof follows closely that of Proposition 4. ■

The definition of the balanced growth path equilibrium of the baseline framework economy concludes this section. It follows closely Definition 3.1, which can be extended to accommodate the baseline framework. Specifically, the definition needs to include the probability matrix (11) and the sequence of creative destruction rates $\{\tau_n\}$.

4 Quantitative Analysis

In this section, I use the theory of provider-driven complementarity to perform a quantitative experiment. The experiment is motivated by the influential contribution of Bloom et al. (2020) arguing that ideas have become harder to find during the last decades. Specifically, I decrease the size of the quality jump obtained after a successful innovation, which is treated as exogenous to the model. This decline can be also be interpreted as a reduction in the probability of obtaining a 'radical' innovation.[22]

I start by considering a version of the model with a mild level of provider-driven complementarity, which I calibrate by targeting average moments of the data between 1985 and 1990. Then I decrease the parameter governing the quality jump's size after a successful innovation (λ). To do that, I explicitly target the decline in the U.S. growth rate between 1985-1990 and 2010-2015. I show that this decline has important effects on business dynamism: the entry rate declines, the concentration of sales increases, and the equilibrium R&D expenditure increases even as the economy's aggregate growth rate declines. To highlight the importance of provider-driven complementarities in explaining the dynamics of firms observed in the data, I conduct a second exercise where I consider a version of the model without provider-driven complementarity. I henceforth refer to this second version of the model as standard quality ladder model. I compare both versions of the model along their respective balanced growth paths with constant rates of growth.

[22] Within this framework, 'radical' innovation is any external innovation (either from an entrant or an incumbent) that allows it to become a market leader. This definition follows that of Acemoglu and Cao (2015).

Contrary to the provider-driven complementarity framework, the standard quality ladder model cannot replicate the evolution of business dynamism observed in the data.

In what follows, I start by briefly describing the numerical solution algorithm and the model's calibration before discussing the results.

4.1 Solution

The solution method is based on computing the equilibrium firm value functions and R&D decisions. This equilibrium is obtained as a fixed point in a space that includes the rate of growth along the economy's balanced growth path, stationary creative destruction rates, and stationary distribution of the number of products. After obtaining the solution for a set of parameters, the model is simulated to obtain the model-based moments of interest.[23]

4.2 Calibration

Here I present the baseline calibration. The model has 11 structural parameters that need to be calibrated, which I partition into two sets. The values of the first set are determined without solving the model by relying on previous literature. The second set of values are determined by solving the model and targeting several moments from the data.

4.2.1 Externally calibrated parameters

I start by setting the discount rate ρ to 2%, a standard value in this literature. The theory I propose relies on the asymmetries generated by provider-driven complementarities across firms of different sizes. As laid out in the previous section, firms' behavior depends on the number of varieties they supply to the market and on the distribution of the number of products across firms. To focus on the effects of provider-driven complementarity, I assume constant returns to scale in the R&D technology for external innovations, i.e., $\sigma = 1 - \psi$, as in (Klette and Kortum, 2004).[24] The parameters $\tilde{\psi}$ and $\hat{\psi}$ govern the curvature of the external and internal R&D cost functions. To assign a value to these parameters, I follow Akcigit and Kerr (2018) that, relying on previous literature, set both equal to 0.5.[25] This implies that both cost functions are quadratic in the Poisson (external or internal) rates of innovation.

The key parameter for the calibration is γ, which measures the strength of provider-driven complementarities. This effect is not trivial to measure in the data, thus given the lack of an estimate for this parameter, I fix a value of $\gamma = 1.0245$, which implies a mild effect of provider-driven complementarity. Later on, I consider alternative values in a sensitivity analysis. As shown in Section 3 with a theoretical example for the simplified model, introducing provider-driven complementarity yields a higher weight of the right tail of the size distribution. With provider-driven complementarity, firms can profit more from each product line obtained through external innovation. Under the assumption of constant returns to scale in the external innovation's production function, the equilibrium

[23]Further details can be found in Appendix A.4.

[24]This assumption has important effects on the relationship between R&D and size. In Appendix A.5.1 I relax this assumption allowing for decreasing returns to scale as in (Akcigit and Kerr, 2018).

[25]This implies that $\psi = 2$.

innovation rates increase with size, leading to bigger firms in equilibrium. It is important to remark that provider-driven complementarity does not play any role in determining the market leader of each variety by assumption in the simplified model. Consequently, the effects of provider-driven complementarity on firm size are reinforced in the baseline framework when provider-driven complementarity interacts with the quality dimension in determining the equilibrium market leader of each variety.

4.2.2 Internally calibrated parameters

It remains to assign parameters to the step size of external and internal innovations, scale parameters of external (for incumbents and entrants) and internal cost functions, and per-product profitability. For simplicity, I assume that the average innovative step obtained from external innovations is the same as the fixed step of internal innovations.[26] This additional restriction leaves me with the following five parameters that need to be determined:

1. Mean value of innovative step size (λ),
2. Scale parameter of external innovation cost function ($\tilde{\chi}$),
3. Scale parameter of internal innovation cost function ($\hat{\chi}$),
4. Scale parameter of entrant cost function (ν),
5. (Inverse) Elasticity of substitution across varieties (β).

Although all the parameters jointly determine the endogenous equilibrium outcomes of interest, each of them is tightly related to a moment of interest. I discuss these connections in what follows.

As is well known, the source of endogenous growth in a quality ladder model is the successful improvement of quality. Therefore, the mean value of innovative step size for external and internal (λ) innovations is closely related to the economy's equilibrium growth rate. The scale parameters of the external and internal innovation cost functions ($\tilde{\chi}$ and $\hat{\chi}$, respectively) will determine the equilibrium expenditure of firms in external and internal R&D. Besides, their interaction determines the ratio of external to internal innovation. Consequently, both scale parameters are connected to the equilibrium ratios of R&D expenditure over total cost and sales. I use my sample of firms from Compustat to target these ratios. The scale parameter of the entrant cost function (ν) determines the entry cost and is closely related to news firms' entry rate. As measuring entry in Compustat is not straightforward, I use data from Business Dynamics Statistics to target the entry rate. Finally, the elasticity of substitution across varieties (β^{-1}) determines the equilibrium per-product profitability of each variety and is closely related to the average profitability of firms. Again, using my Compustat sample of firms, I target the average ratio of total sales minus total operating expenditures before depreciation to total sales.

Based on the previously established connections, I initially calibrate the model to the average moments of interest between 1985 and 1990. Specifically, I compute model implied

[26] In terms of the parameters shown in previous sections, that implies $\tilde{\lambda}_x = \lambda_z \equiv \lambda$.

moments and compare them to their counterpart moments in the data. Technically, I solve the simple minimum distance equation given by

$$\min \sum_{j=1}^{5} \frac{|\text{model}(j) - \text{data}(j)|}{|\text{data}(j)|}.$$

4.2.3 Fit of the Model

Table 1 shows the full set of calibrated parameter values for the provider-driven complementarity model and the standard quality ladder model as well as the fit of both models to the targeted moments.

Table 1: Calibrated parameters

Parameter	PDC	ST	Target	Data	PDC	ST
λ	0.113	0.112	GDPpc growth (%)	2.5	2.5	2.5
$\tilde{\chi}$	4.914	4.59	(Mean) R&D / Sales (%)	6.4	6.8	6.2
$\hat{\chi}$	0.593	0.592	(Mean) R&D / Total Cost (%)	7.4	7.8	7.2
ν	0.799	0.823	Entry Rate (%)	11.9	11.9	11.9
β	0.195	0.198	Mean profitability (%)	12.5	12.5	13.4

Note. PDC: Provider-driven complementarity model features. ST: Standard quality ladder model. Second and third columns report values for the internally calibrated parameters. Values of the externally calibrated parameters: $\rho = 0.02$, $\psi = 0.5$, $\sigma = 0.5$, $\gamma = 1.0245$ (PDC), $\gamma = 1$ (ST).

Both models do an excellent job in replicating the targeted moments for the 1985-1990 period. Introducing a mild provider-driven complementarity effect does not turn into a significant change in the parameters' values with respect to those of the standard quality ladder model. Moreover, the values of the parameters I find are similar to those of Akcigit and Kerr (2018), even though they use a different sample of firms. In what follows, I provide intuition on how the parameters differ across the provider-driven complementarity framework and the standard quality ladder model.

According to the predictions of the theory highlighted in Section 3, under provider-driven complementarity, incumbent firms have an extra incentive to innovate - both externally and internally - to increase their profits. Consequently, to be able to match the desired ratios between R&D and total cost or total sales, a lower cost to conduct R&D is needed in the standard quality ladder model. In turn, this implies that in the standard quality ladder model, the scale parameters of the external ($\tilde{\chi}$) and internal ($\hat{\chi}$) innovation cost functions are slightly lower than their counterparts in the provider-driven complementarity model. The calibrated values imply that around 55% of total growth comes from external innovation, while 20% comes from internal innovation (the remaining 25% is attributed to entry). These results are comparable to those of Akcigit and Kerr (2018). However, in their recent contribution, Garcia-Maza et al. (2019) show that up to 70% of growth can stem from internal innovation. In Appendix A.5.2 I show that my main results are preserved even adjusting the calibration to capture that alternative relationship between external and internal R&D. The theory I propose in this paper also implies that entrants are less likely to become market leaders as they do not generate provider-driven complementarity upon entry while incumbents do. Consequently, the calibrated value of the scale parameters of the entrant cost function (ν) needed to obtain the entry rate of firms observed in the data must be smaller under provider-driven complementarity.

The growth rate in both models is 2.5%, the same as observed in the data. By construction, the source of endogenous growth in a quality ladder model is the successful improvement of quality. In particular, the economy grows as additional successful rungs of the quality ladder are attained. The differences in the needed step sizes of innovation (λ) across models are minor but highlight an important equilibrium outcome: under provider-driven complementarity, there are fewer active firms in equilibrium, and incumbents tend to be bigger. This is why, although incumbent firms innovate more in this framework, a bigger step size of innovation is needed to obtain the growth rate observed in the data.

Finally, both models closely match the average profitability of firms found in the data. It is important to note that the ratio between profits and sales in the model includes R&D expenditures. Consequently, a relatively higher value is needed under provider-driven complementarity compared to the value obtained for the standard quality ladder model. One relevant remark about profitability in the model is that, by construction, markups are exogenous and fixed. Therefore, the model would only generate big changes in firms' profitability if markups were endogenous or, at least, time-varying. This would be reflected in the model through exogenous changes in the (inverse) elasticity of substitution across varieties β. Although I abstract from these changes in the main exercise, recent literature has pointed out that markups have indeed changed during the last decades, and are important to explain recent macroeconomic trends, see for example Barkai (2020), De Loecker et al. (2020) or Feijoo Moreira (2020). In Appendix A.5.3 I show that my main results are preserved even if an increasing trend in markups is introduced into the model.

4.3 Results

I now discuss the implications of a decline in the 'radicalness' of innovation on business dynamism, which is the quantitative analysis's primary objective. This exercise is motivated by the literature arguing that ideas have become harder to find during the last decades, which I capture in the model by reducing the average innovation step size (λ) obtained after a successful innovation.

To perform the exercise, I re-calibrate λ to match the observed decline in the U.S. growth rate between 1985-1990 and 2010-2015, leaving the remaining parameters constant. This delivers a value of λ equal to 0.073 with provider-driven complementarity and 0.072 in the standard quality ladder model. In what follows, I compare both models along their respective balanced growth paths. The results of the quantitative exercise are summarized in Table 2.

The first row of Table 2 shows that both models can accommodate the decline in the economy's growth rate, reproducing the observed decline in the data closely. From (16) it follows that reducing the quality jump, everything else constant, decreases the increase in quality upon successful innovation therefore reducing the growth rate of the economy.

The remaining rows of Table 2 show the effects of the decline in the other moments of interest, which are all non-targeted. The decline in the innovative step size generates opposite effects across both versions of the model in:

Table 2: Quantitative analysis Results. Changes in the U.S. between 1985-1990 and 2010-2015.

Moment	Data	PDC	ST
GDPpc growth (p.p.)	-0.99	-1.00	-0.99
(Mean) R&D / Sales (p.p.)	3.71	0.97	-0.16
(Mean) R&D / Total Cost (p.p.)	4.32	1.01	-0.17
Entry Rate (p.p.)	-2.89	-0.28	0.06
Top 20% Sales (p.p.)	1.68	1.52	-2.19

Note. PDC: Baseline provider-driven complementarity model. ST: Standard quality ladder model.

1. the equilibrium R&D expenditure relative to the total sales or total cost of firms,
2. the equilibrium entry rates,
3. the share of sales accounted for by the 20% biggest firms of the economy.

To provide intuition on the reasons underlying this opposite behavior and help better understand the general equilibrium effects of the experiment, in what follows, I start by discussing the implications of the decline within the standard quality ladder model. This model is a particular case of the baseline framework when $\gamma = 1$, thus many of its implications carry over to the provider-driven complementarity framework. In Figure 9 I show a battery of theoretical results for both models, which I now turn to discuss in detail.

4.3.1 Standard quality ladder model

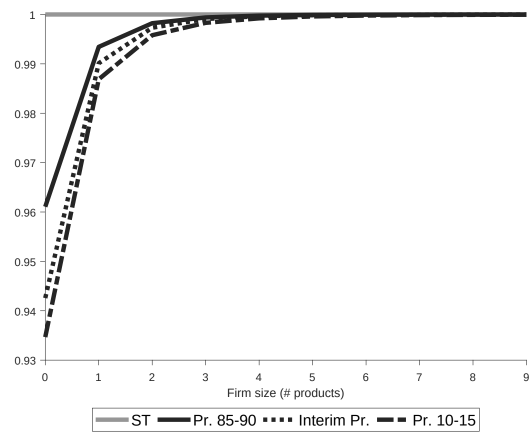
Consider the effects of the decline in the step size of innovation for the standard quality ladder model. Figure 9c shows that when the innovation step size declines, the value function's coefficient capturing the stream of discounted profits of being a market leader increases (ST 85-90 vs. ST 10-15). This is a direct consequence of the decline in the economy's growth rate, which drives down the general equilibrium interest rate according to the household's Euler equation (1). I henceforth refer to this result as the *market effect*, which enhances the incentives to conduct R&D. Moreover, firm profitability is also determined by its quality portfolio. When the innovative step size declines, a firm expects to obtain smaller quality improvements over after a successful innovation (internal or external), which hinders its incentives to conduct R&D. I henceforth refer to this result as the *quality effect*.

In Figure 9d I show that the equilibrium per-product internal innovation rates decline sharply, implying that the *quality effect* dominates the *market effect*. This is an intuitive result: internal R&D allows increasing the quality of a variety the firm already produces, i.e., a firm conducting internal R&D is already the market leader of that variety. As a consequence, the marginal benefit of increasing the quality of that variety is dominated by the expected quality increase after innovating.[27] As the average innovative step size

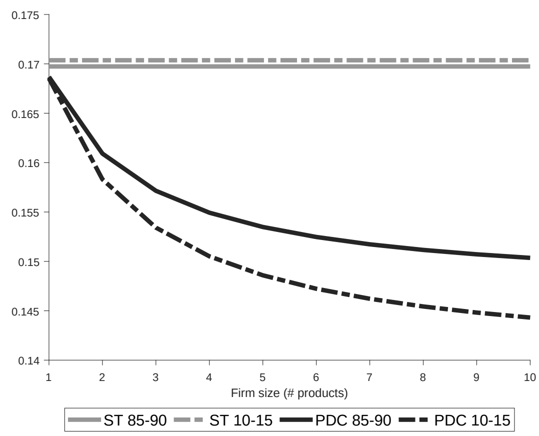
[27] Loosely speaking, the productivity of investing in internal R&D declines, and the marginal profit gain is now lower.

Figure 9: Quantitative analysis results

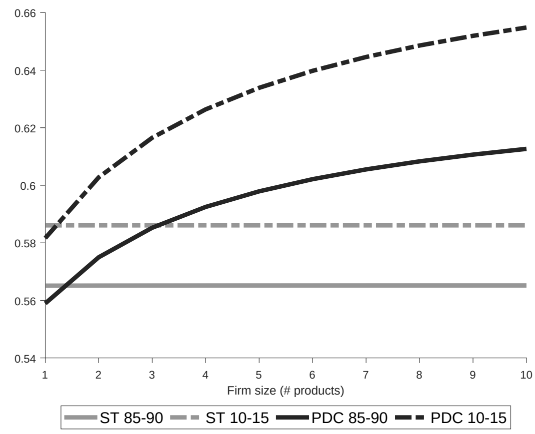
(a) Probability successful external R&D



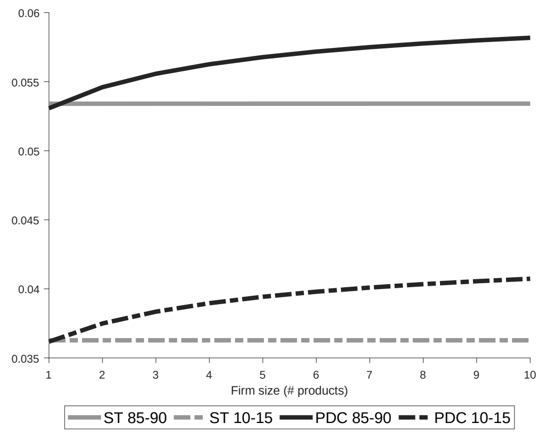
(b) Creative destruction rate



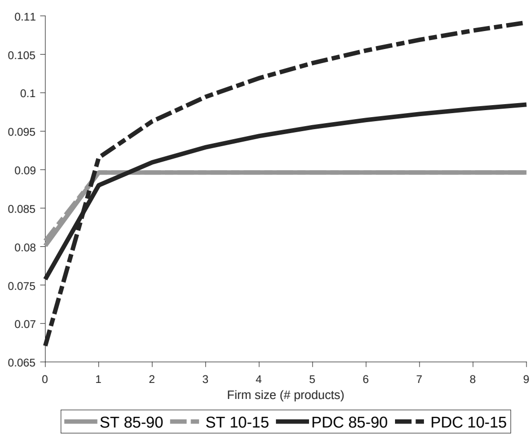
(c) Value function A_n coefficient



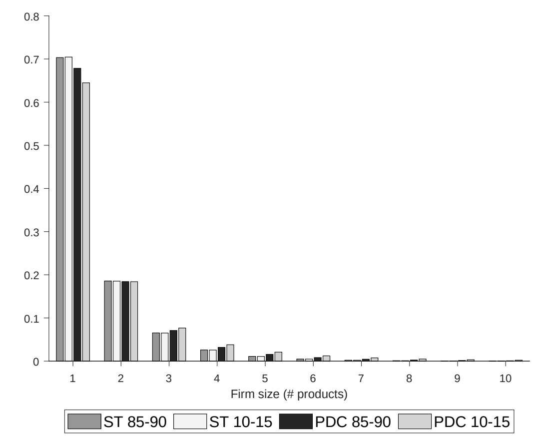
(d) Per-product internal innovation rate



(e) Per-product external innovation rate



(f) # products distribution



Note. PDC: provider-driven complementarity model. ST: standard quality ladder model. Note: along all panels, x-axis represents number of products - 0 refers to a potential entrant. Panel 9a represents the expected probability of replacing a incumbent through external innovation conditional on the innovator current size. Interim Pr. depicts the probability of replacing a incumbent after declining the step size innovation, but fixing the firm size distribution to its 1985-1990 level.

declines, the incentive to conduct internal R&D also does. This naturally leads to less internal R&D in equilibrium, reducing R&D expenditure.

The dynamics of external R&D, represented in Panel 9e, are substantially different. After the innovative step size decline, the per-product external innovation rates are barely affected. Opposite to the internal R&D decision, a firm conducts external R&D to become the market leader of a variety outside its quality portfolio. As a consequence, for the external R&D decision, the increase in the stream of discounted profits associated with being a market leader (the *market effect*) is now as important as the decline in the expected quality increase after innovating (the *quality effect*). Provided that the per-product external innovation rates remain constant, both effects cancel out for incumbent firms. However, for potential entrants, I find that the *market effect* slightly dominates the *quality effect*, leading to a small increase in the innovation rate of entrants. This increase has three important implications: i) it leads to an increase in the equilibrium entry rate (see 2); ii) it generates a small increase in the rate of creative destruction of the economy, as shown in Figure 9b; and iii) it yields an increase in the measure of incumbent firms in equilibrium. Finally, Figure 9f shows that the distribution of the number of products across firms slightly compresses. As the distribution of the number of products across firms and the size distribution of firms are tightly linked,[28] it follows that the share of sales of the biggest firms in the economy declines as shown in Table 2.

4.3.2 Provider-driven complementarity

Before discussing the implications of provider-driven complementarity, it is relevant to highlight that in this framework the probability of obtaining a successful innovation is a function of size, as Figure 9a shows. This is opposed to the standard quality ladder model, where this probability is constant and equal to one, i.e., any successful quality improvement always find its way to the market. According to (3), as firms increase their portfolio, they generate a higher level of complementarity. Therefore small firms, all else equal, find it more challenging to become market leaders as they provide less complementarity to the consumer. When ideas get harder to find, more quality innovations that would be successful in the absence of provider-driven complementarity do not find their way into the markets. Although the decline in the innovative step size reduces the ex-ante probability for any firm of the economy, its effects are more relevant for small firms. In other words, all firms are less likely to become market leaders, but small firms are relatively more unlikely.

The decline in the probability of obtaining a successful innovation can be separated into two components. The first one is purely mechanical: reducing the average step size innovation implies that firms are less likely to obtain a successful innovation even in the absence of general equilibrium effects. The second component that affects the decline in the probability of obtaining a successful innovation is the change in the firm size distribution, which is a general equilibrium outcome. Ultimately, the change in the firm size distribution alters the level of complementarity offered by every firm in equilibrium. The dotted line in Panel 9a (Interim Pr.) is intended to capture the first component by representing the counterfactual probability of obtaining a successful innovation after

[28] This is a common feature of this type of models which is also highlighted in Akcigit and Kerr (2018) or Cavenaile and Roldan-Blanco (2019).

the decline in the innovative step size but had remained the firm size distribution as in the period 1985 - 1990. The dashed line (Pr. 10-15) represents the actual probability obtained in the new equilibrium in 2010 - 2015. Thus the difference between the dotted and the dashed line captures the second component. An important and direct equilibrium effect generated by the probability of obtaining a successful innovation can be observed in the rate of creative destruction represented in Panel 9b, which becomes decreasing in firm size. As a firm becomes the market leader of more markets, it generates a higher provider-driven complementarity effect. In turn, this becomes a barrier to entry for smaller firms: it becomes more unlikely that a big firm loses a product through creative destruction.

Most of the effects of the decline in the innovative step size in the standard quality ladder model are preserved under provider-driven complementarity. Specifically, Panel 9c shows that the stream of discounted profits of being a market leader also increases; however, under provider-driven complementarity, it additionally increases in firm size. The increase can be separated into two parts. First, there is a symmetric effect across all firms stemming from the equilibrium interest rate decline, precisely as in the standard quality ladder model. Second, a non-linear equilibrium effect is inherited from the decline in the creative destruction rate following the decline in the innovation step size. In other words, as big firms suffer a lower rate of creative destruction, their discounted stream of benefits increases as they expect to remain longer as market leaders. The decline in the innovation step size affects internal R&D in the same way as in the standard quality ladder model, the *quality effect* still dominates the *market effect*.

Under provider-driven complementarity, the effects on external R&D are different from those obtained from the standard-quality ladder model. On the one hand, as the probability that an entrant obtains a successful innovation declines sharply, this drives down the incentives to conduct R&D for entrants. On the other hand, the incentives to conduct R&D for an incumbent increase and are also increasing in firm size, following the non-linear increase in the profitability of being an incumbent. In other words, for an entrant, the *quality effect* dominates the *market effect*, while the opposite happens for an incumbent. The asymmetric effect of provider-driven complementarities between entrants and incumbents is key for the results. In particular, entrants do not generate provider-driven complementarity, which implies that besides finding it more difficult to become market leaders, upon entry they are also more likely to lose their market leadership due to creative destruction. Incumbents, in turn, generate provider-driven complementarity, which not only increases the profitability of all their product lines but drives down the probability of losing a product through creative destruction.

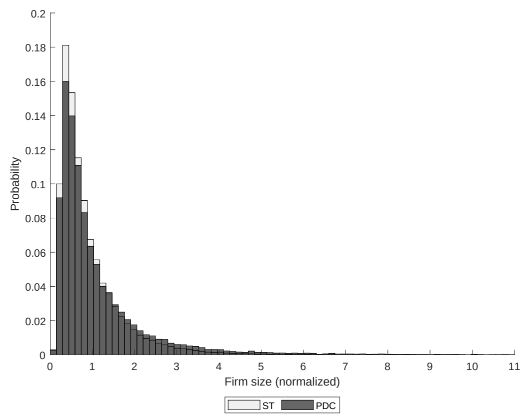
In summary, the standard quality ladder model predicts a decline in internal innovation R&D rates, accompanied by small, almost negligible, effects on external R&D rates. As a consequence, R&D expenditure declines. In contrast, under provider-driven complementarity, the increase in external R&D innovation rates compensates for the decline in internal R&D, leading to an increase in R&D expenditure. As highlighted before, the decline in the innovation step size also affects firms' equilibrium size distribution. In the standard quality ladder model, the small increase in entrants' innovation rate leads to an increase in new firms' entry rate and the equilibrium measure of incumbent firms. Finally, the firm size distribution slightly compresses, which yields a decline in the concentration of sales. Under provider-driven complementarity, I find the opposite. Specifically, the strong decline in the entrants innovation rate leads to a decline in new firms' entry rate, as shown in Table 2. Specifically, the mechanism proposed in this paper accounts for roughly a 10% of the decline in the entry rate observed in the data. This decline and the increase in the external R&D innovation rates of incumbents lead to a decline in the number of incumbents in equilibrium. Moreover, as 9f shows the firm's size distribution shifts to the right, implying that a substantial share of firms become bigger in equilibrium. In turn, this implies an increase in the concentration of sales as shown in Table 2.

4.4 Non-targeted moments

The quantitative experiment results show the importance of the asymmetry between entrants and incumbents generated by provider-driven complementarity. In this Subsection, I highlight the performance of the model along several non-targeted dimensions.

As it is well known in the literature, the standard quality ladder model generates tails of the distribution slimmer than the data. Figure 10 represents the simulated invariant firm size distribution combined with the quality margin. The provider-driven complementarity framework improves upon the standard quality ladder by generating a thicker right tail of the sale distribution, although the tails of the distribution are still not as fat as in the data.

Figure 10: Firm size distribution, standard quality ladder vs. provider-driven complementarity



Note: Normalized quality relative to average quality.

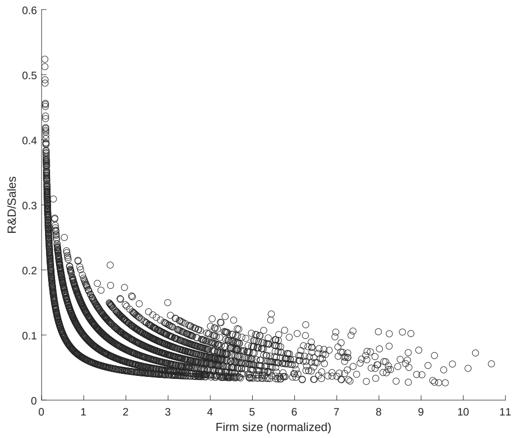
Finally, I review the relationship between R&D intensity and firm size. Recent literature has documented that smaller innovative firms usually exhibit higher R&D intensity on average.[29] The preceding Subsection shows that bigger firms conduct higher per-product innovation rates (both internal and external) under provider-driven complementarity, even under the assumption of constant returns to scale in the production of external innovations. However, the relationship between R&D intensity and firm size is ultimately determined by the growth of R&D expenditure relative to the growth of sales as the size of firms increase.

Figure 11 shows that the provider-driven complementarity framework correctly predicts the declining relationship between R&D intensity and firm size. Recall that provider-

[29] See, among others, Akcigit and Kerr (2018), Cavenaile and Roldan-Blanco (2019).

driven complementarity is assumed to be increasing in firm size. From (6) the optimal quantity supplied is proportional to the level of provider-driven complementary, thus intermediate sales also are. As the (normalized) firm size increases, the declining relationship between R&D intensity and firm size disappears, resulting in constant returns to scale in the external innovation production function.

Figure 11: Relationship between R&D/Sales and firm size



Note: Normalized quality relative to average quality.

4.5 Sensitivity

In this section, I report on a series of sensitivity exercises regarding the strength of provider-driven complementarity. First, to stress the interaction between innovative step size and provider-driven complementarity and its effects on the creative destruction rate, I conduct the baseline exercise using the simplified model in the first sensitivity exercise. In the second exercise, I show how sensitive the main experiment results are to changes in γ, the parameter governing the strength of provider-driven complementarity.

I start by describing the results of the simplified framework. To conduct this sensitivity exercise, I recalibrate all the model's parameters following the same procedure as before, and I conduct the same exercise based on declining the step size of innovation. In the simulation of the model, this step size is assumed to be constant and bigger than the maximum possible value of provider-driven complementarity γ. The results are shown in the first part of Table 3.

In the simplified framework, the effect of provider-driven complementarity is always offset by a successful quality increase. This implies that potential entrants, small firms and big firms are equally likely to obtain a successful innovation that allows them to become the market leader of a variety. In other words, provider-driven complementarity is irrelevant in determining the market leader of each variety. Consequently, provider-driven complementarity affects firms' decisions and outcomes through its effect on the return function, but it does not affect the rate of creative destruction experienced by firms of different sizes. This ultimately implies that the industrial organization of firms is not relevant for the individual decisions of firms. As a consequence, the results of the simplified model are both qualitatively and quantitatively very close to the predictions of the standard quality ladder model, which is in line with the theoretical predictions laid out in Section 3. Although there is marginally more entry as the step size innovation declines in the simplified framework, the concentration of sales declines less than in the standard quality ladder model. This is given by the higher skewness of the firm size distribution of firms in the simplified framework, a feature highlighted in Section 3. Therefore, even under more entry, the simplified model can sustain a higher level of concentration. This highlights that the key mechanism driving the baseline provider-driven complementarity model results is the interaction of the quality dimension with the complementarity dimension.

Table 3: Sensitivity Analysis Results. Changes in the U.S. between 1985-1990 and 2010-2015.

Simplified vs. Baseline

Moment	ST	Simplified	PDC
GDPpc growth (p.p.)	−0.99	−0.99	−1.00
(Mean) R&D / Sales (p.p.)	−0.16	−0.15	0.97
(Mean) R&D / Total Cost (p.p.)	−0.17	−0.16	1.01
Entry Rate (p.p.)	0.06	0.05	−0.28
Top 20% Sales (p.p.)	−2.19	−2.07	1.52

Sensitivity with respect to γ

Moment	PDC	$\gamma = 1.0215$	$\gamma = 1.0275$
GDPpc growth (p.p.)	−1.00	−0.99	−1.00
(Mean) R&D / Sales (p.p.)	0.97	0.70	1.32
(Mean) R&D / Total Cost (p.p.)	1.01	0.73	1.38
Entry Rate (p.p.)	−0.28	−0.17	−0.38
Top 20% Sales (p.p.)	1.52	0.59	2.55

Note. The results for the standard-quality ladder model (ST) in the first part and the provider-driven complementarity model (PDC) in the second are obtained from Table 2 and represented here for comparative purposes. In the simplified model the value of γ is 1.0245 as in the baseline provider-driven complementarity model (PDC).

The second sensitivity exercise is based on the specification of γ, the parameter governing provider-driven complementarity. Now I report on the sensitivity of the main experiment results to changes in the value of this parameter, which can be understood as varying the strength of provider-driven complementarity. As it was highlighted in the model's calibration, the effect of provider-driven complementarity is not readily measurable in the data. To avoid this complication, I assumed a mild level of provider-driven complementarity by fixing a value $\gamma = 1.0245$. Now I consider the alternative values $\gamma = 1.0215$ and $\gamma = 1.0275$. In each case, I recalibrate all the model's parameters following the same procedure as before and conduct the same exercise based on declining the step size of successful innovation. The results are shown in the second part of Table 3. As the strength of provider-driven complementarity declines (increases), the increase in the equilibrium R&D expenditure is smaller (bigger), the entry rates declines less (more), and the share of sales of the biggest firms increases less (more). In summary, the absolute value of the moments of interest is monotone in the value of γ.

It can be shown that as $\gamma \to 1$, the results converge to those of the standard quality-ladder model. Interestingly, this implies that for low levels of provider-driven complementarity, it is possible to obtain qualitatively aligned results with those of the standard quality ladder model. This is not surprising as, as stressed before, the interaction between quality and complementarity in determining each market leader is key for the results. If γ declines, even very small quality improvements can be enough to offset the provider-driven complementarity effect, independently of the incumbent's size. As a consequence, as γ declines, provider-driven complementarity becomes less important and ultimately negligible.

5 Conclusion

This paper explores the role of provider-driven complementarity to explain several salient facts about declining business dynamism. In particular, the main interest lies in increasing R&D expenditures and concentration yet decreasing entry rates and economic growth that have taken place during the last two decades.

I propose a theory where provider-driven complementarity makes seemingly independent products become complements when provided by a single firm. In particular, I build on the Akcigit and Kerr (2018) quality ladder model of endogenous growth through R&D, which I extend to explore the effects of this kind of complementarity. The model remains tractable and allows closed-form solutions. The main difference between a standard quality ladder model and the provider-driven complementarity model is that complementary acts as a barrier to entry. In the absence of product complementarities, successful R&D improving the quality of any good enables the innovating firm to displace the previous lower-quality incumbent. However, when firms generate provider-driven complementarity, consumers do not necessarily switch to the state-of-the-art higher quality product but may remain attached to the lower-quality incumbent if the product complementarity effect is sufficiently large. Therefore, small firms need to develop sufficiently 'radical' quality improvements so that the products they produce can find their way into the markets.

I then conduct a quantitative experiment motivated by the recent literature arguing that ideas have become harder to find during the last decades. I show that a mild level of provider-driven complementarity can speak to declining business dynamism within this environment. In particular, as innovation gets less radical and the economy's growth rate declines, the model predicts an increase in R&D expenditure given by the reaction of incumbents, which ultimately yields a decline in the entry rate of new firms and an increase in the concentration of sales. I show that a standard quality ladder model without provider-driven complementarity yields the reverse predictions.

It is worth noting that the theory developed here does not assume limit-pricing, an otherwise frequent assumption in quality ladder models. Limit-pricing opens the door for markup heterogeneity, a widely documented fact in the data. However, allowing for limit-pricing under provider-driven complementarities gives rise to complicated although exciting and novel markup dynamics beyond this paper's scope. Additionally, the theory does not allow for mergers and acquisitions between firms. However, the provider-driven complementarity is a suitable framework to explain the increased number of mergers and acquisitions observed in the data. Both issues are left for future research.

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A Appendix

A.1 Data

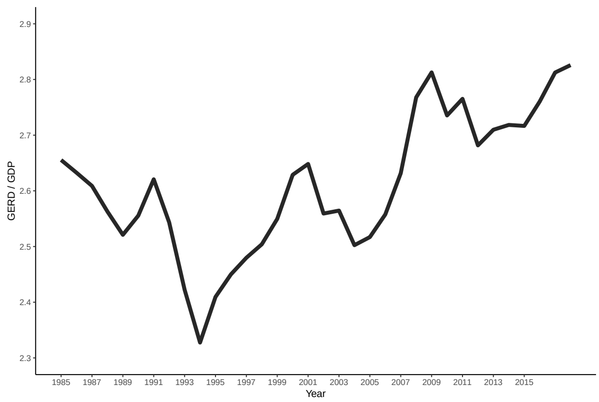
A.1.1 Data arrangements

I perform the following series of adjustments in the sample of firms obtained from Compustat. First, I restrict the sample to firms that are observed for at least five consecutive years. Second, I restrict my attention to firms with positive sales that report values of Cost of Goods Sold (COGS) and Selling, Administrative and General Expenditure (SGA). Even though I am mainly interested in firms that conduct Research and Development (R&D) activities, I do not exclude firms from the sample that do not conduct R&D. However; I exclude firms that do not report R&D expenditure. I also exclude firms that report a level of R&D expenditure such that its share of R&D as a function of operating expenditure (the sum of COGS and SGA) is above 1.

A.1.2 Aggregate data on R&D expenditure

The increase in R&D can also be observed by using aggregate data from OECD. The next Figure shows that the share of gross domestic expenditure on R&D has increased during the last decades.

Figure 12: Share of GDP of Gross domestic expenditure on R&D



Note: Gross domestic expenditure on R&D (GERD) includes expenditure on research and development by business enterprises, higher education institutions, as well as government and private non-profit organisations. Source: OECD.

A.2 Proofs and derivations

A.2.1 Proposition 1

Proof. For any distribution of products across firms $\{\mu_n\}_{n\geqslant 1}$, it is always possible to find a sequence of real numbers $\{i_1, i_2, \ldots\}$, such that the $[0, 1]$ product space can be arranged in a way that varieties $j \in [0, i_1)$ are produced by firms with $n = 1$; $i \in [i_1, i_2)$ are produced by firms with $n = 2$; $\ldots$; and, finally, $i \in [i_{N-1}, 1)$ are produced by very big firms with

$n = N$ and $N \to \infty$. Therefore, one can write

$$\int_0^1 m_j q_j \, \mathrm{d}j = \lim_{N\to\infty} \sum_{n=1}^{N} \gamma^{1-\frac{1}{n}} \int_{i_{n-1}}^{i_n} q_j \, \mathrm{d}j, \tag{28}$$

where $i_0 = 0$, $\lim_{n\to\infty} i_n = 1$, and

$$\bar{q}_n = \int_{i_{n-1}}^{i_n} q_j \, \mathrm{d}j$$

denotes the average quality of each subset of products. Following the same argument laid out in the main text, each subset grows at an expected rate

$$g_n = F\mu_n(\lambda_z n z_n + \lambda_x n \tau_n),$$

As the equilibrium satisfies an invariant distribution of firms (see Proposition 6), the implied provider-driven complementarity effect also is. This implies that along the balanced growth path the only source of growth is the increase in quality through R&D. Therefore the law of large numbers assures that the sum in (28) asymptotically grows at the same rate as that of average quality, g.

■

A.2.2 Value Functions

Consider an innovator that upon entry would have quality portfolio of size $i \geqslant 1$ if the incumbent in a given variety has a quality portfolio of size $j \geqslant 1$. Re-define the indicator function that control which innovations are successful in replacing the incumbent as

$$\xi_{i,j} \equiv \mathbf{1}_{\{\Lambda_x \geqslant \gamma^{\Delta(i,j)}\}}.$$

For the Simplified framework, $\xi_{i,j} = 1, \forall i, j$. Bellman's principle states that value function for an incumbent firm of size $1 \leqslant n = |\mathbf{q}_i|$ at some point in time $\bar{t}$ can be obtained as

$$\begin{aligned}
V_{n,\bar{t}-\Delta t}(\mathbf{q}_i, \bar{q}_{\bar{t}-\Delta t}) = \max_{\substack{x_{n,\bar{t}-\Delta t},\\ \{z_{jn,\bar{t}-\Delta t}\}_{j=1}^n}} & \Bigg\{ \sum_{q_j \in \mathbf{q}_i} \left[\pi_{\bar{t}-\Delta t}\gamma^{1-n^{-1}} - \hat{\chi} z_{jn,\bar{t}-\Delta t}^{\hat{\psi}}\right] q_j \Delta t - \tilde{\chi} x_{n,\bar{t}-\Delta t}^{\tilde{\psi}} n^{\tilde{\sigma}} \bar{q}_t \Delta t \\
& + e^{-r_{\bar{t}}\Delta t}\Bigg(\sum_{q_j \in \mathbf{q}_i} \Big[z_{jn,\bar{t}-\Delta t}\Delta t V_{n,\bar{t}}(\mathbf{q}_i \setminus_- \{q_j\} \cup_+ \{q_j\Lambda_z\}, \bar{q}_{\bar{t}}) \\
& + \tau_{n,\bar{t}-\Delta t}\Delta t V_{n-1,\bar{t}}(\mathbf{q}_i \setminus_- \{q_j\}, \bar{q}_{\bar{t}}) \\
& + x_{n,\bar{t}-\Delta t}\Delta t \mathbb{E}_{h,\lambda_x}\left[\xi_{n+1,j}\left\{V_{n+1,\bar{t}}(\mathbf{q}_i \cup_+ \{q_h\Lambda_x\}, \bar{q}_{\bar{t}}) - \Omega_n q_j\right\}\right] \Big] \\
& + \Big(1 - \sum_{q_j \in \mathbf{q}_i} \left[z_{jn,\bar{t}-\Delta t} + \tau_{n,\bar{t}-\Delta t} + x_{n,\bar{t}-\Delta t}\right]\Delta t\Big) V_{n,\bar{t}}(\mathbf{q}_i, \bar{q}_{\bar{t}}) \Bigg) \Bigg\}
\end{aligned}$$

where to ease notation I avoid including second or higher order terms in the Poisson arrival rates for internal and external innovation or creative destruction rates. Performing a first-order Taylor expansion of the continuation value yields

$$V_{n,\bar{t}}(\mathbf{q}_i, \bar{q}_{\bar{t}}) + \Bigg[- r_{\bar{t}} V_{n,\bar{t}}(\mathbf{q}_i, \bar{q}_{\bar{t}}) + \sum_{q_j \in \mathbf{q}_i} \Big[z_{jn,\bar{t}} V_{n,\bar{t}}(\mathbf{q}_i \backslash_{-} \{q_j\} \cup_{+} \{q_j \Lambda_z\}, \bar{q}_{\bar{t}})$$

$$+ \tau_{n,\bar{t}} V_{n-1,\bar{t}}(\mathbf{q}_i \backslash_{-} \{q_j\}, \bar{q}_{\bar{t}}) + x_{n,\bar{t}} \mathbb{E}_{h,\lambda_x} \left[\xi_{n+1,j} \left\{ V_{n+1,\bar{t}}(\mathbf{q}_i \cup_{+} \{q_h \Lambda_x\}, \bar{q}_{\bar{t}}) - \Omega_n q_j \right\} \right] \Big) \Bigg] \Delta t.$$

Substituting this expression in the original value function, it remains to divide by Δt both sides, and take limits as $\Delta t \to 0$ to obtain

$$r_{\bar{t}} V_{n,\bar{t}}(\mathbf{q}_i, \bar{q}_{\bar{t}}) = \max_{x_{n,\bar{t}},\ \{z_{nj,\bar{t}}\}_{j=1}^n} \Bigg\{ \sum_{q_j \in \mathbf{q}_i} \Big[\pi_{\bar{t}} \gamma^{1-n^{-1}} q_j + \tau_{n,\bar{t}} \big[V_{n-1,\bar{t}}(\mathbf{q}_i, \bar{q}_{\bar{t}} \backslash_{-} \{q_j\}) - V_{n,\bar{t}}(\mathbf{q}_i, \bar{q}_{\bar{t}}) \big]$$

$$+ z_{nj,\bar{t}} \big[V_{n,\bar{t}}(\mathbf{q}_i \backslash_{-} \{q_j\} \cup_{+} \{q_j \Lambda_z\}, \bar{q}_{\bar{t}}) - V_{n,\bar{t}}(\mathbf{q}_i, \bar{q}_{\bar{t}}) \big] - \hat{\chi} z_{nj,\bar{t}}^{\hat{\psi}} q_j \Big]$$

$$+ n x_{n,\bar{t}} \mathbb{E}_{h,\lambda_x} \Bigg[\mathbf{1}_{\{\Lambda_x \geqslant \gamma^{\Delta(n+1,j)}\}} \Big[V_{n+1,\bar{t}}(\mathbf{q}_i \cup_{+} \{q_j \Lambda_x\}, \bar{q}_{\bar{t}}) - V_{n,\bar{t}}(\mathbf{q}_i, \bar{q}_{\bar{t}}) - \Omega_n \sum_{q_j \in \mathbf{q}_i} q_j \Big] \Bigg]$$

$$- \tilde{\chi} x_{n,\bar{t}}^{\tilde{\psi}} n^{\tilde{\sigma}} \bar{q}_{\bar{t}} \Bigg\}$$

A.2.3 Proposition 3

Proof. Plugging the guess

$$V_n(\mathbf{q}_i, \bar{q}) = A_n \sum_{q_j \in \mathbf{q}_i} q_j + B_n \bar{q},$$

in the value function of an incumbent yields[30]

$$rA_n \sum_{q_j \in \mathbf{q}_i} q_j + rB_n \bar{q} = \max_{x_n,\ \{z_j\}_{j=1}^n} \Bigg\{ \pi \gamma^{1-n^{-1}} \sum_{q_j \in \mathbf{q}_i} q_j + \sum_{q_j \in \mathbf{q}_i} z_{nj} A_n q_j \lambda_z - \hat{\chi} \sum_{q_j \in \mathbf{q}_i} z_{nj}^{\hat{\psi}} q_j$$

$$+ \tau \Big[(n-1) A_{n-1} \sum_{q_j \in \mathbf{q}_i} q_j - n A_n \sum_{q_j \in \mathbf{q}_i} q_j + n(B_{n-1} - B_n) \bar{q} \Big]$$

$$+ n x_n \Big[(A_{n+1} - A_n) \sum_{q_j \in \mathbf{q}_i} q_j + A_{n+1} \bar{q} \Lambda_x + (B_{n+1} - B_n) \bar{q} \Big]$$

$$- x_n^{\tilde{\psi}} n^{\tilde{\sigma}} \bar{q} - n x_n (A_{n+1} - A_n) \sum_{q_j \in \mathbf{q}_i} q_j \Bigg\} + B_n \bar{q} g$$

which trivially reduces to

$$rA_n \sum_{q_j \in \mathbf{q}_i} q_j + rB_n \bar{q} = \max_{x_n,\ \{z_j\}_{j=1}^n} \Bigg\{ \pi \gamma^{1-n^{-1}} \sum_{q_j \in \mathbf{q}_i} q_j \sum_{q_j \in \mathbf{q}_i} z_{nj} A_n q_j \lambda_z - \hat{\chi} \sum_{q_j \in \mathbf{q}_i} z_{nj}^{\hat{\psi}} q_j$$

[30] Note that

$$\dot{V}_n(\mathbf{q}_i, \bar{q}) = B_n \dot{\bar{q}} = B_n \bar{q} g.$$

$$+\tau\Bigg[(n-1)A_{n-1}\sum_{q_j\in\mathbf{q}_i} q_j - nA_n\sum_{q_j\in\mathbf{q}_i} q_j + n(B_{n-1}-B_n)\bar{q}\Bigg]$$

$$+ nx_n\left[A_{n+1}\bar{q}\Lambda_x + (B_{n+1}-B_n)\bar{q}\right] - \tilde{\chi}x_n^{\tilde{\psi}}n^{\tilde{\sigma}}\bar{q}\Bigg\} + B_n\bar{q}g$$

On the one hand, collecting terms with $\bar{q}$ yields

$$(r-g)B_n = n\tau(B_{n-1}-B_n) + \max_{x_n}\left\{nx_n[A_{n+1}\Lambda_x + B_{n+1} - B_n] - \tilde{\chi}x_n^{\tilde{\psi}}n^{\tilde{\sigma}}\right\}. \tag{29}$$

The FOC that characterizes external innovation of a firm with size $n \geqslant 1$ is given by

$$n(A_{n+1}\Lambda_x + B_{n+1} - B_n) - \tilde{\psi}\tilde{\chi}x_n^{\tilde{\psi}-1}n^{\tilde{\sigma}} = 0,$$

where rewriting yields

$$x_n = \left(\frac{A_{n+1}\Lambda_x + B_{n+1} - B_n}{\tilde{\psi}\tilde{\chi}n^{\tilde{\sigma}-1}}\right)^{\frac{1}{\tilde{\psi}-1}}.$$

Define

$$\mathcal{A} \equiv A_{n+1}\Lambda_x + B_{n+1} - B_n,$$

Substituting x_n in (29) yields

$$(r-g)B_n = n\tau(B_{n-1}-B_n) + n\frac{\mathcal{A}^{\frac{1}{\tilde{\psi}-1}+1}}{\left(\tilde{\psi}\tilde{\chi}n^{\tilde{\sigma}-1}\right)^{\frac{1}{\tilde{\psi}-1}}} - \tilde{\chi}\left(\frac{\mathcal{A}}{\tilde{\psi}\tilde{\chi}n^{\tilde{\sigma}-1}}\right)^{\frac{\tilde{\psi}}{\tilde{\psi}-1}} n^{\tilde{\sigma}}, \tag{30}$$

and simplifying the RHS gives

$$\begin{aligned}
(r-g)B_n &= n\tau(B_{n-1}-B_n) + \mathcal{A}^{\frac{\tilde{\psi}}{\tilde{\psi}-1}}\left[n\left(\frac{1}{\tilde{\psi}\tilde{\chi}n^{\tilde{\sigma}-1}}\right)^{\frac{1}{\tilde{\psi}-1}} - \tilde{\chi}n^{\tilde{\sigma}}\left(\frac{1}{\tilde{\psi}\tilde{\chi}n^{\tilde{\sigma}-1}}\right)^{\frac{\tilde{\psi}}{\tilde{\psi}-1}}\right] \\
&= n\tau(B_{n-1}-B_n) + (\tilde{\psi}-1)\tilde{\chi}n^{\tilde{\sigma}}\left(\frac{A_{n+1}\Lambda_x + B_{n+1} - B_n}{\tilde{\psi}\tilde{\chi}n^{\tilde{\sigma}-1}}\right)^{\frac{\tilde{\psi}}{\tilde{\psi}-1}} \\
&= n\tau(B_{n-1}-B_n) + \frac{(\tilde{\psi}-1)}{\tilde{\psi}^{\frac{\tilde{\psi}}{\tilde{\psi}-1}}\tilde{\chi}^{\frac{1}{\tilde{\psi}-1}}n^{\frac{\tilde{\sigma}-\tilde{\psi}}{\tilde{\psi}-1}}}\left(A_{n+1}\Lambda_x + B_{n+1} - B_n\right)^{\frac{\tilde{\psi}}{\tilde{\psi}-1}}.
\end{aligned}$$

Rewriting yields

$$B_{n+1} = B_n - A_{n+1}\Lambda_x + \tilde{\psi}\tilde{\chi}^{\frac{1}{\tilde{\psi}}}n^{\frac{\tilde{\sigma}-\tilde{\psi}}{\tilde{\psi}}}\left(\frac{(\rho+n\tau)B_n - n\tau B_{n-1}}{\tilde{\psi}-1}\right)^{\frac{\tilde{\psi}-1}{\tilde{\psi}}}.$$

On the other hand, collecting terms with q_j yields

$$rA_n = \pi\gamma^{1-n^{-1}} + \max_{\{z_{nj}\}}\left\{z_{nj}A_n\lambda_z - \hat{\chi}z_j^{\hat{\psi}}\right\} + \tau\left[(n-1)A_{n-1} - nA_n\right]. \tag{31}$$

The FOC that characterizes internal innovation is given by

$$A_n\lambda_z - \hat{\chi}\hat{\psi}z_{nj}^{\hat{\psi}-1} = 0,$$

where rewriting yields

$$z_{nj} = \left(\frac{A_n \lambda_z}{\hat{\chi}\hat{\psi}}\right)^{\frac{1}{\hat{\psi}-1}}.$$

Substituting this expression in (31) obtain

$$(r + \tau n)A_n = \pi\gamma^{1-n^{-1}} + \hat{\chi}(\hat{\psi} - 1)\left(\frac{A_n \lambda_z}{\hat{\chi}\hat{\psi}}\right)^{\frac{\hat{\psi}}{\hat{\psi}-1}} + \tau(n-1)A_{n-1}.$$

■

A.2.4 Proposition 4

Proof. By induction, I first check it holds for $n = 1$ and $n = 2$. For $n = 1$

$$\mu_1 = \frac{x_e}{F\tau_1},$$

which is the same condition obtained from the first flow equation. For $n = 2$

$$\mu_2 = \frac{1}{2}\frac{x_e}{Fx_2}\frac{x_1 x_2}{\tau_1\tau_2} = \frac{1}{2}\frac{x_e}{F}\frac{x_1}{\tau_1\tau_2},$$

where re-arranging and adding and subtracting $x_e\tau_1$ in both sides yields

$$F\mu_2 2\tau_2\tau_1 + x_e\tau_1 = x_e(x_1 + \tau_1).$$

Dividing through by τ_1 write

$$F\mu_2 2\tau_2 + x_e = \frac{x_e}{\tau_1}(x_1 + \tau_1) = F\mu_1(x_1 + \tau_1),$$

which is the same condition obtained from the second flow equation.

Now suppose it holds for n and $n-1$, and prove it also holds for $n+1$. In this case

$$\mu_{n-1} = \frac{1}{n-1}\frac{x_e}{Fx_{n-1}}\prod_{s=1}^{n-1}\frac{x_s}{\tau_s},$$

and

$$\mu_n = \frac{1}{n}\frac{x_e}{Fx_n}\prod_{s=1}^{n}\frac{x_s}{\tau_s}.$$

From the third flow equation write

$$F\mu_{n+1}(n+1)\tau_{n+1} + F\frac{1}{n-1}\frac{x_e}{Fx_{n-1}}\prod_{s=1}^{n-1}\frac{x_s}{\tau_s}(n-1)x_{n-1} = F\frac{1}{n}\frac{x_e}{Fx_n}\prod_{s=1}^{n}\frac{x_s}{\tau_s}n(\tau_n + x_n),$$

where simplifying, re-arranging and multiplying and dividing in the RHS by x_{n+1} yields

$$\mu_{n+1} = \frac{1}{n+1}\frac{x_e}{Fx_{n+1}}\prod_{s=1}^{n+1}\frac{x_s}{\tau_s}.$$

■

A.3 Extension: Limit-pricing

In this Appendix I explore an alternative environment that relaxes Assumption 1. In particular, I extend the model by allowing for limit-pricing and endogenous markups, in a similar fashion as in (Peters, 2018). This Section shows, under very restrictive assumptions, how to introduce provider-driven complementarity in an otherwise standard limit-pricing framework of quality ladder models.

Consider the final good production function (as opposed to (2))

$$\ln Y_t = \int_0^1 \ln \left[\left(\sum_{i \in \mathcal{F}} [m_{ijt} q_{ijt} k_{ijt}]^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \right] \mathrm{d}j, \tag{32}$$

where for simplicity labor is not a factor of production. Assuming $\theta = \infty$ implies that, once adjusted by quality and provider-driven complementarity, different vintages of each variety j supplied by different firms $i \in \mathcal{F}$ are perfect substitutes. This implies that only the variety with the highest adjusted-quality to price ratio will be demanded in equilibrium. We can therefore reduce the previous expression to

$$\ln Y_t = \int_0^1 \ln \left(m_{jt} q_{jt} k_{jt} \right) \mathrm{d}j.$$

Normalizing $P_t^Y = 1$, $\forall t$, and dropping time subscripts (just to ease notation), the solution to the final good producer problems yields the inverse demand function

$$p_{jt} = \frac{Y_t}{k_{jt}}.$$

Assume production of intermediates is linear in labor, that is $k_{jt} = l_{jt}$, thus the marginal cost of production is the wage rate w_t and is constant across all varieties. Furthermore, assume that intermediate producers compete à la Bertrand. As (32) exhibits unitary demand elasticity of substitution, these assumptions imply that it is optimal for the market leader will to limit-price its closest competitor. Suppose that the current market leader in variety j has size n_j and can produce variety j with quality q_j, while its closest follower has size $n_{-j}$and can produce variety j with quality q_{-j}. The markup set by the market leader of variety j is

$$\mu_{jt} = \frac{\gamma^{1-\frac{1}{n_j}}}{\gamma^{1-\frac{1}{n_{-j}}}} \frac{q_j}{q_{-j}} = \gamma^{\Delta(n_j, n_{-j})} \Lambda_j,$$

where $\Delta(n_j, n_{-j}) \equiv (n_j - n_{-j})/n_j n_{-j}$ and $\Lambda_j \equiv q_j/q_{-j}$. This implies that the equilibrium price of variety j is

$$p_{jt} = \gamma^{\Delta(n_j, n_{-j})} \Lambda_j w_t.$$

Finally, per-variety profits are given by

$$\pi_{jt}(\Delta(n_j, n_{-j}); \Lambda_j) = p_{jt} k_{jt} - w_t k_{jt} = (p_{jt} - w_t) k_{jt} = \left(1 - \frac{1}{\gamma^{\Delta(n_j, n_{-j})} \Lambda_j} \right) Y_t.$$

Consequently, the market leader of each product needs to keep track of the size of its closest follower and its quality, as both jointly determine the equilibrium price and ultimately the profitability of each product. Note that contrary to the general framework analyzed in the main text, any change in a follower's size leads to a change in the equilibrium markup. Consider the following example. Suppose that there are 3 firms in the economy, denoted as A, B and C, and a total of 6 varieties. Without loss of generality, suppose that firm A is the market leader of 4 varieties, B is the market leader of the remaining 2, and C is a potential entrant. Now, suppose that C obtains a successful innovation over one of the products of B and becomes the market leader in that variety. In the baseline framework, the profitability of firm A would not change - only its incentives to conduct R&D through due to the change in the firm size distribution. However, with limit pricing, profitability changes, as now a follower has 1 product less, which implies that A can now charge a higher price. This example highlights that creative destruction outcomes with limit-pricing and provider-driven complementarity are much richer, but more complicated than in the baseline model.

In the remainder of this Section, I characterize the value function of an incumbent firm. I start by assuming that the innovation technology is now characterized by a general cost function $\Gamma(\{x_n, z_{nj}\}_{j=1}^n)$ where x_n and z_{nj} are the per-product external and internal innovation rates. If successful, quality improves as in the general framework, i.e. a firm can produce any randomly drawn product (not previously owned) with incremental quality $q_{jt+\Delta t} = q_{jt}(1+\tilde{\lambda}_x) \equiv q_{jt}\Lambda_x$, with $\tilde{\lambda}_x > 0$ drawn from an exponential distribution with parameter λ_x^{-1}. This allows to characterize the probability in the exact same way as in the general framework. Incumbent firms are subject to a rate of creative destruction τ_n which is size-dependent, and is also defined as in the general framework. Finally, I assume that the market leader of each variety can also improve the quality distance with respect to its closest competitor by investing in internal R&D. If successful, this firm can produce variety j with quality $q_{jt+\Delta t} = q_{jt}(1+\tilde{\lambda}_x) \equiv q_{jt}\Lambda_x$, with $\tilde{\lambda}_x > 0$, so that the quality jump with respect to its closest competitor is now given by $\Lambda_j(1+\lambda_x)$.

The state of a size n incumbent firm and is determined by its size, the collection of size gaps with respect to each of its followers $\mathbf{\Delta} \equiv \{\Delta(n_j, n_{-j})\}_{j=1}^n = \{\Delta n_{-j}\}_{j=1}^n$,[31] and the corresponding quality jump in each variety $\mathbf{\Lambda} \equiv \{\Lambda_j\}_{j=1}^n$. Following the notation laid out in the characterization of the baseline model, the value function of such an incumbent is now given by

$$\begin{aligned}
rV_n(\mathbf{\Delta};\mathbf{\Lambda}) = & \sum_{j=1}^{n} \pi_j(\Delta n_{-j};\Lambda_j) \\
& + \sum_{j=1}^{n} \tau_n \Bigg[V_{n-1}\big(\mathbf{\Delta}\backslash_{-}\{\Delta(n_{-j})\};\mathbf{\Lambda}\backslash_{-}\{\Lambda_j\}\big) - V_n\big(\mathbf{\Delta};\mathbf{\Lambda}\big) \\
& \quad + \sum_{i\neq j} \Bigg(V_{n-1}\big(\mathbf{\Delta}\backslash_{-}\{\Delta n_{-j}\} \cup_{+} \{\Delta n_{-j} - 1\};\mathbf{\Lambda}\big) - V_n\big(\mathbf{\Delta};\mathbf{\Lambda}\big) \Bigg) \Bigg] \\
& + \sum_{j=1}^{n} \tau_{n_{-j}} \Big[V_n\big(\mathbf{\Delta}\backslash_{-}\{\Delta n_{-j}\} \cup_{+} \{\Delta n_j + 1;\mathbf{\Lambda}\}\big) - V_n\big(\mathbf{\Delta};\mathbf{\Lambda}\big) \Big] \\
& + \sum_{j=1}^{n} n_{-j}\tilde{x}_{n_{-j}} \mathbb{E}_i\left[\xi_{n_{-j}+1,i}\right] \Big(V_n\big(\mathbf{\Delta}\backslash_{-}\{\Delta n_{-j}\} \cup_{+} \{\Delta n_{-j} - 1;\mathbf{\Lambda}\}\big) - V_n\big(\mathbf{\Delta};\mathbf{\Lambda}\big) \Big)
\end{aligned}$$

[31] In the latter notation, $\Delta n_{-j} = n - n_{-j}$, i.e., a firm only needs to track the its size difference with its closest competitor in each variety.

$$
\begin{aligned}
&+ \max_{\{z_{nj},x_n\}_{j=1}^n} \Bigg\{ \sum_{j=1}^{n} z_{nj} \Big(V_n\big(\mathbf{\Delta}; \Lambda\backslash_{-}\{\Lambda_j\} \cup_{+} \{\Lambda_j(1+\lambda_x)\}\big) - V\big(\mathbf{\Delta};\mathbf{\Lambda}\big)\Big) \\
&\quad + n x_n \Big(\mathbb{E}_i \Big[\xi_{n+1,i} \Big(V_{n+1}\big(\widetilde{\mathbf{\Delta}} \cup_{+} \{\Delta n_{-i}\}; \mathbf{\Lambda} \cup_{+} \{\Lambda_i\}\big)\Big)\Big] - V_n\big(\mathbf{\Delta};\mathbf{\Lambda}\big)\Big) \\
&\quad - \Gamma(\{x_n, z_{nj}\}_{j=1}^n) \Bigg\} + \dot{V}_t\big(\mathbf{\Delta};\mathbf{\Lambda}\big)
\end{aligned}
$$

where

$$
\xi_{n,i} \equiv \Pr\left(1+\lambda_x \geqslant \gamma^{\Delta(n,i)}\right),
$$

and

$$
\widetilde{\mathbf{\Delta}} \equiv \mathbf{\Delta}\backslash_{-}\{\Delta n_{-j}\}_{j=1}^n \cup_{+} \{\Delta n_{-j}+1\}_{j=1}^n.
$$

The value of a firm with size n and portfolio $(\mathbf{\Delta},\mathbf{\Lambda})$ is characterized by the following elements. The first line captures the instantaneous profits for each variety in which the firm is the outstanding market leader. The second and third lines capture two direct effects of creative destruction on the value of the firm. On the ne hand, the second line captures the standard loss of a variety through creative destruction. On the other hand, the third line captures the fact that the firm must re-adjust its limiting-price in all the remaining varieties after that loss, as now it generates less provider complementarity. In this framework, creative destruction also affects the firm's profitability through the effects on the products of the incumbents. In this sense, the fourth and fifth lines capture two indirect effects of creative destruction. Specifically, the fourth captures the firm's increase in profits if any of the followers in each product line loses one product due to creative destruction of a third firm. Moreover, the fifth line represents the decline in the firm's profits if any of the followers in each product line increases its product portfolio at the expense of a third firm. The remaining three lines characterize the R&D decisions of the firm. The sixth line captures the change in value if internal innovation is successful. The sixth line captures the increase in value due to external innovation. If the firm's R&D is successful, it improves the quality of any product i and becomes the new market leader of that variety. Moreover, it can increase its profits in all the remaining product lines as now it generates more complementarities. Given that R&D is undirected, the quality of the new product is unknown and is captured by the expected value $\mathbb{E}_i$, which is is an expectation over varieties –which determine the size of the current market leader in variety i– and innovation step Λ_i. The quality jump has to be sufficiently big to offset the incumbent's provider-driven complementarity to become the new producer. The probability term again captures this.

The analysis of the value function highlights that this framework allows for richer firm dynamics, but it hinders the tractability of the model. I leave this interesting avenue for further research.

A.4 Solution algorithm

I solve the generalized framework as a nested fixed point over the balanced growth path rate of the economy, along which the equilibrium relationship between $\bar{q}$ and $\bar{Q}$ remains constant. The algorithm follow the next steps:

1. Guess $M > 1$ such that along the balanced growth path, $\bar{Q} = M\bar{q}$.
   (a) Guess a growth rate g.
      i. Guess a firm size distribution and a sequence of creative destruction rates $\{\tau_n\}$.
      ii. Given the distribution, characterize the probability sequence of successful innovation, i.e., the probability of a firm of each size to obtain a successful innovation conditional on the distribution of incumbents.
      iii. Solve for the sequences $\{A_n\}$, $\{B_n\}$, $\{x_n\}$ and $\{z_n\}$ in Proposition 5.[32]
      iv. Compute the implied firm-size distribution and creative destruction rates. If they are the same as the initial guess, go to the next step. If not converged, go back to step i. by using as new guess a linear interpolation between the old and new guess.
   (b) Compute the implied growth rate of the economy, if not converged, go back to (a) and update the guess until convergence.
2. Simulate an economy and verify that $\bar{Q} = M\bar{q}$, otherwise, go back to 1. and update the guess.

To simulate the model, I assume an economy composed by 50,000 varieties, with initial quality normalized to 1. Moreover, I assume that initially, each good is produced by a single firm. I simulate the economy for 2,000 periods of length $\Delta t = 0.1$. As time is continuous and the number of varieties is kept constant, at each simulated instant, I draw a vector of shocks that determines whether there could be entry in a new variety (a new firm obtains a quality innovation for this variety), external innovation (an incumbent in a different variety obtains a quality innovation) or internal innovation (the incumbent obtains a quality innovation). If any of the two first events happens, a quality jump is drawn from the exponential distribution, and the new market leader on the variety is chosen.[33] Over time, this fixed size economy converges to its balanced growth path where all aggregate varieties grow at the rate g and the firm size distribution is constant.

A.5 Robustness

In this Appendix, I perform a series of alternative experiments to the baseline quantitative analysis carried out in Section 4. Specifically, I recalibrate all the model's parameters following the same procedure as described in the main text, and I conduct the same exercise based on declining the step size of innovation.

A.5.1 Returns to scale in external R&D

I start by reviewing the relationship between $\psi > 0$ and $\sigma > 0$ in the external R&D production function, which determines the returns to scale the production function. In

[32]The critical step is solving for the sequence $\{B_n\}$. As provider-driven complementarities vanish asymptotically, and the external R&D cost function will be assumed to have constant returns to scale, as n grows, B_n must converge to a line.

[33]In the unlikely event of ties, equal probabilities are assigned to both firms, and a coin is tossed to decide the market leader.

the baseline experiment I impose $\psi + \sigma = 1$, so that the production function exhibits constant returns to scale as in (Klette and Kortum, 2004). In this Section, I choose $\psi + \sigma =< 1$ so that it exhibits decreasing returns to scale as in (Akcigit and Kerr, 2018). Specifically, the value of ψ remains as in the baseline exercise but following (Akcigit and Kerr, 2018) I fix a value of $\sigma = 0.395$. The results are summarized in Table 4.

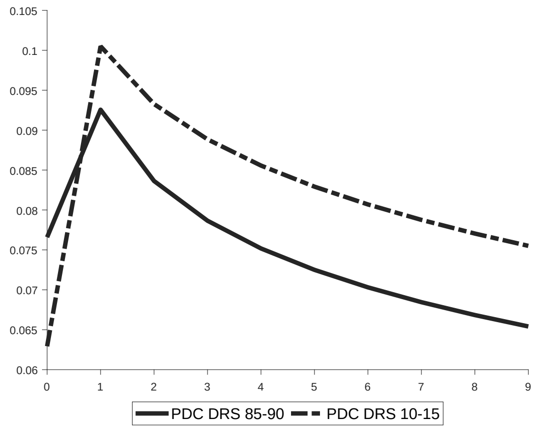
Allowing for decreasing returns to scale in the external innovation production function does not change the qualitative nature of the baseline quantitative analysis results. In the absence of provider-driven complementarity, any assumption regarding the cost function's curvature yields the same results. However, with decreasing returns to scale, the innovative step size decline is strengthened if firms generate provider-driven complementarity. The reasons underlying these results are the same as in the baseline experiment, i.e., when the innovative step size declines, the probability of obtaining a successful innovation also does. As potential entrants do not generate provider-driven complementarity until their product portfolio grows, they now find it more challenging to become market leaders. Under decreasing returns to scale, it is more expensive to grow through external R&D upon entry. Therefore, a stronger effect of provider-driven complementarity is needed to obtain comparable results to the baseline. In that case, even though big incumbents are subject to a relatively higher cost, they find that compensated through the effect of provider-driven complementarity, which increases the profitability of all their product lines and drives down the probability of losing a product through creative destruction for an incumbent firm.

Table 4: Robustness: Returns to scale in R&D production function.

	Baseline		DRS	
Moment	PDC	ST	PDC	ST
GDPpc growth (p.p.)	−1.00	−0.99	−0.98	−1.00
(Mean) R&D / Sales (p.p.)	0.97	−0.16	1.24	−0.19
(Mean) R&D / Total Cost (p.p.)	1.01	−0.17	1.30	−0.18
Entry Rate (p.p.)	−0.28	0.06	−0.64	0.07
Top 20% Sales (p.p.)	1.52	−2.19	2.12	−2.29

Note. DRS: Decreasing returns to scale. PDC: Provider-driven complementarity model. ST: Standard quality ladder model. Columns 1 and 2 are obtained from Table 2.

Figure 13: Per-product external innovation rates with decreasing returns to scale



Note: x-axis represents number of products - 0 denotes a potential entrant.

It is instructive to look at the profile of per-product innovation rates as a function of size, represented in Figure 13. As the product portfolio of a firm increases, the probability of losing a product through creative destruction declines, which increases the profitability of being an incumbent. Consequently, even if the external innovation production function exhibits decreasing returns to scale when the innovative step size declines, bigger firms increase their external innovation rates relatively more with respect to smaller firms. As a final remark, in the main text, I show that the provider-driven complementarity framework correctly predicts the declining relationship between R&D intensity and firm size, even assuming constant returns to scale. As under decreasing returns to scale, the innovation rates decline with size, this relationship is only reinforced.

A.5.2 Relationship between internal and external R&D

The baseline quantitative analysis's calibrated values imply that around 55% of total growth comes from external innovation, while 20% comes from internal innovation and 25% is due to the entry of new firms. While these results are comparable to those of Akcigit and Kerr (2018), in this Section, I consider an alternative calibration where internal R&D is the main source of growth, accounting for 60% of total growth. This is similar to the results of Garcia-Maza et al. (2019), which show that even up to 70% of growth can stem from internal innovation.

Table 5: Robustness: Relationship between internal and external R&D.

	Baseline		Ext. < Int.	
Moment	PDC	ST	PDC	ST
GDPpc growth (p.p.)	−1.00	−0.99	-1.00	−0.99
(Mean) R&D / Sales (p.p.)	0.97	−0.16	0.68	−0.38
(Mean) R&D / Total Cost (p.p.)	1.01	−0.17	0.71	−0.40
Entry Rate (p.p.)	−0.28	0.06	−0.10	0.27
Top 20% Sales (p.p.)	1.52	−2.19	2.53	−1.21

PDC: Provider-driven complementarity model. ST: Standard quality ladder model. Columns 1 and 2 are obtained from Table 2.

Table 4 shows that the qualitative nature of the results is preserved. Relative to the baseline exercise, when internal innovation is the main channel of firm growth, the decline in the innovative step size of innovation generates a stronger increase in the entry rate in the standard quality ladder model and a milder decline in the entry rate in the provider-driven complementarity framework. This happens because in this exercise conducting external R&D becomes relatively more expensive (than in the baseline), which favors potential entrants relative to active incumbents.

Focusing on the provider-driven complementarity framework, as the innovative step size declines the probability of obtaining a successful innovation declines and an incumbent firm's profits increase through the same mechanism described in the main text. However, incumbent firms react by increasing less their external R&D rates as it is relatively costlier now. This observation has two important implications. On the one hand, the firms' equilibrium distribution shifts less to the right than in the baseline exercise, i.e., firms become bigger, but less so than in the baseline. Consequently, although probability of obtaining a successful innovation declines, it declines less than in the baseline, specifically for potential entrants, so that the entry rate declines less than in the baseline. On the other hand, the R&D expenditure of firms increases less than in the baseline. Interestingly, even though quantitatively, these results are milder than in the baseline, the concentration of sales in the economy increases. This happens because in this economy, firms conduct more internal R&D than in the baseline. Therefore, even though the distribution of firms shifts less to the right, big firms in this alternative economy can concentrate a higher share of the total sales of the economy as they innovate more in internal R&D, which ultimately implies that on average bigger firms concentrate a larger amount of higher quality, more complementary – and ultimately, more expensive – products.

A.5.3 Increasing trend in markups

In the baseline quantitative exercise, firms' markups are assumed to be constant. However, recent papers as Barkai (2020), De Loecker et al. (2020) or Feijoo Moreira (2020) show that markups have increased during the last decades and are important to explain recent macroeconomic trends. In this Section, I consider an alternative calibration where markups increase by 25% between 1985-1990 and 2010-2015. The results are summarized in Table 6.

Introducing an increasing trend in markups simultaneously that the step size of innovation declines reinforces the results obtained in the baseline framework. This happens because in this economy, the profits associated with being an incumbent increase not only as a consequence of the decline in the growth rate of the economy – and thus the equilibrium interest rate – but also because now firms can charge higher markups. Consequently, incumbent firms have a much higher incentive to conduct external R&D to increase their quality portfolio. However, as in the baseline exercise the reaction of potential entrants is different between the standard quality ladder model and the provider-driven complementarity framework. In the standard quality model, potential entrants invest relatively more in R&D to obtain market leadership. Together with the increase in the external R&D rates of incumbents, the aggregate expenditure in R&D increases. Since there are more firms in the economy, the concentration of sales declines. In contrast, the opposite happens in the provider-driven complementarity framework. The strong increase in the external R&D of incumbents increases not only the creative destruction rate suffered by small firms relative to big firms, but also drops down the probability of obtaining a successful innovation. The consequence is a strong decline in the entry rate of new firms in the economy. Moreover, this increasing R&D effort of incumbents is reflected in a stronger increase in R&D expenditure. Finally, the combination of declining entry and the shift to the right of the size distribution implies an increase in sales concentration.

Table 6: Robustness: Declining step size of innovation with increasing trend in markups

	Baseline		Increasing	
Moment	PDC	ST	PDC	ST
GDPpc growth (p.p.)	−1.00	−0.99	−0.99	−1.00
(Mean) R&D / Sales (p.p.)	0.97	−0.16	1.55	0.69
(Mean) R&D / Total Cost (p.p.)	1.01	−0.17	2.17	0.23
Entry Rate (p.p.)	−0.28	0.06	−1.31	2.55
Top 20% Sales (p.p.)	1.52	−2.19	2.98	−3.41

PDC: Provider-driven complementarity model. ST: Standard quality ladder model. Columns 1 and 2 are obtained from Table 2.